Microfoundations of DSGE Models: III Lecture

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Contents

- A New Keynesian model with capital accumulation
- A New Keynesian model with capital accumulation, habit persistence and adjustment costs on labour and investments
- A New Keynesian model with capital accumulation and rule-of-thumb households
- A *complete* model

Model #1: A NK Model with Capital Accumulation

Main Features

- Agents
  - a continuum of households that consume, own capital and supply differentiated labour services;
  - a continuum of trade unions each of which representing workers of a certain type;
  - a continuum of intermediate goods producers that employ labour inputs, rent capital from households and produce differentiated intermediate goods (monopolistic competition);
  - a continuum of final goods producers that use intermediate goods to produce a homogenous final good consumed by households (perfect competition);
  - the government that set public spending;
  - the central bank that implements monetary policy.
- Imperfect competition
- Prices rigidities
- Business cycles driven by nominal and real shocks
- Rational expectations and no asymmetric information

Model #1: A NK Model with Capital Accumulation
Final-Good Firms

Each firm takes the other firms' prices as given.

The representative final-good firm uses $Y_t(j)$ units of each intermediate good $j \in [0, 1]$ purchased at a nominal price $P_t(j)$ to produce Y_t units of the final good with a constant returns to scale technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}$$

$\theta =$ the elasticity of substitution across intermediate goods, $\theta > 1$. As $\theta \to \infty$ higher and higher degree of substitution $\to$ less market power of intermediate-goods producers.

Model #1: A NK Model with Capital Accumulation
Final-Good Firms

The problem of the representative firm is to max their profits wrt $Y_t(j)$ with $j \in [0, 1]$ (static problem) given the available technology.

$$P_t Y_t - \int_0^1 P_t(j) Y_t(j) \, dj$$

Profit maximization yields the following set of demands for intermediate goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

Perfect competition and free entry drives the final good-producing firms' profits to zero, so that from the zero-profit condition we obtain:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}.$$

which defines the aggregate price index of the economy and is such that $P_t Y_t = \int_0^1 P_t(j) Y_t(j) \, dj$.

Model #1: A NK Model with Capital Accumulation
Intermediate-Goods Firms

Each intermediate good j is produced by a monopolist which has a production function of the form:

$$Y_t(j) = A_t L_t(j)^\alpha K_t(j)^{1-\alpha}$$

where $0 < \alpha < 1$

A_t = Total factor productivity, $A_t = A \exp(\varepsilon_{A,t})$ and $\varepsilon_{A,t} = \rho_A \varepsilon_{A,t} + \xi_{A,t}$ with $\xi_{A,t} \sim iid.N(0, \sigma_A^2)$

$L_t(j)$ = CES aggregate of labor inputs supplied by unionized workers defined below (see below)

$K_t(j)$ = physical capital

Model #1: A NK Model with Capital Accumulation
Price Rigidities à la Rotemberg

According to Rotemberg (1983) each monopolistic firm faces a quadratic cost of adjusting nominal prices, measured in terms of the final-good:

$$ADJ\_P_t(j) = \frac{\gamma_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t$$

where $\gamma_p =$ degree of nominal price rigidities.
Firm j will not always choose to charge the optimal price (i.e. $P_t^*(j) = \frac{\theta}{\theta-1} MC_t(j)$) since it is assumed to face convex costs to changing its price.

Model #1: A NK Model with Capital Accumulation
Price Rigidities à la Rotemberg

Why are price changes costly?

- There is the cost of physically changing posted prices (probably a fixed cost per price change).
- A firm that changes its prices may face a cost which results from the negative reaction of its costumers (reputation loss). From this point of view, probably customers react more strongly to large price changes than to gradual variations.

Model #1: A NK Model with Capital Accumulation

Price Rigidities à la Rotemberg

- What is the main difference between the Calvo price setting and the Rotemberg adjustment costs hypothesis?
  - In the Rotemberg model there is no price dispersion. In each period t firm j can change its price price $P_t(j)$ s.t. the payment of the adjustment cost. All firms face the same problem and will set the same price and produce the same quantity of each differentiated good (symmetry).
  - In the Calvo model there's price dispersion. Firms will not set the same price (asymmetry).
- Different types of inefficiency in the two models
  - In the Rotemberg model the cost of nominal rigidities consists in a wedge between aggregate demand ($C_t + I_t + G_t$) and aggregate output (Y_t), since a fraction of the output goes in the price adjustment cost.
  - In the Calvo model, nominal rigidities through price dispersion, makes aggregate output less efficient.

Model #1: A NK Model with Capital Accumulation
Price Rigidities à la Rotemberg

Despite the economic difference between these two models of price rigidities, up to a first order approximation and around a zero inflation steady state, they imply the same reduced form of the New Keynesian Phillips curve.

Otherwise, the Rotemberg model seems to be more robust to non-linearities (implying more robust results).

The implications of of having trend inflation in the two pricing models.

On these issues: see Ascari and Merkl (2009); Ascari and Ropele (2007); Ascari and Rossi (2009, 2010).

Model #1: A NK Model with Capital Accumulation
Intermediate-Goods Firms

Given the wage index W_t and the rental rate of capital r_t^k, the problem for firm j is to choose $\{L_t(j), K_t(j), P_t(j)\}_{t=0}^{\infty} = 0$ in order to maximize the sum of expected discounted real profits

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^R}{\lambda_0^R} \left\{ \begin{array}{c} \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} L_t(j) - r_t^k K_t(j) + \\ -ADJ\_P_t(j) \end{array} \right\},$$

given $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$ and where
$ADJ\_P_t(j) = \frac{\gamma_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$.

Model #1: A NK Model with Capital Accumulation
Intermediate-Goods Firms

To solve firm's j problem, consider the Lagrangian function

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^R}{\lambda_0^R} \left\{ \begin{array}{c} \left[\begin{array}{c} \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} L_t(j) - r_t^k K_t(j) \\ -\frac{\gamma_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \end{array} \right] + \\ -MC_t(j) \left[Y_t(j) - A_t L_t(j)^\alpha K_t(j)^{1-\alpha} \right] \end{array} \right\}$$

where $MC_t(j) =$ real marginal cost.

Model #1: A NK Model with Capital Accumulation
Intermediate-Goods Firms

Using $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$ we have:

FOC wrt $P_t(j)$

$$\left[\frac{1}{P_t}(1-\theta) + MC_t(j)\theta\frac{1}{P_t(j)}\right] Y_t(j) = \frac{\partial ADJ\_P_t(j)}{\partial P_t(j)} +$$
$$+\beta E_t \frac{\lambda_{t+1}^R}{\lambda_t^R}\frac{\partial ADJ\_P_{t+1}(j)}{\partial P_t(j)}$$

Remark: for $\gamma_p = 0$ under symmetry: $MC_t = \frac{\theta-1}{\theta}$, that is $P_t = \frac{\theta}{\theta-1} MC_t^N$.

Model #1: A NK Model with Capital Accumulation
Intermediate-Goods Firms

FOC wrt $K_t(j)$

$$r_t^k = (1 - \alpha) MC_t(j) A_t L_t(j)^\alpha K_t(j)^{-\alpha}$$

FOC wrt $L_t(j)$

$$\frac{W_t}{P_t} = \alpha MC_t(j) A_t L_t(j)^{\alpha-1} K_t(j)^{1-\alpha}$$

Model #1: A NK Model with Capital Accumulation
Households and Preferences

Consider a continuum of households index by $i \in [0, 1]$. Household i is characterized by the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{i,t}) + \frac{\omega}{1-v} \left(1 - L_{i,t}\right)^{1-v} \right]$$

$C_{i,t}$ consumption; $L_{i,t}$ specific labour inputs; β is the time discount factor; $v < 1$

The period-by-period budget constraint is

$$
\begin{aligned}
P_t C_{i,t} + B_{i,t} + P_t I_{i,t} = {}& W_{i,t} L_{i,t} + (1 + r_{t-1}) B_{i,t-1} + \\
& + P_t r_t^K K_{i,t} + D_{i,t} - P_t TAX_{i,t}
\end{aligned}
$$

where $I_{i,t} = K_{i,t+1} - (1 - \delta) K_{i,t}$; $D_{i,t} =$ dividends; $W_{i,t} =$ nominal wage; $TAX_t =$ lump-sum taxes; $r =$ nominal interest rate; $B_{i,t-1} =$ nominal bonds issued by the government.

Model #1: A NK Model with Capital Accumulation
Households and Preferences

The representative household will choose $\{C_{i,t}, B_{i,t}, K_{i,t+1}\}_{t=0}^{\infty}$ so as to max the lifetime utility function given the sequence of budget constraint. To solve household's j problem, consider the Lagrangian function

$$\mathcal{L}_0^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \log\left(C_{i,t}\right) + \frac{\omega}{1-v}\left(1 - L_{i,t}\right)^{1-v} + \\ \lambda_{i,t} \left[\begin{array}{c} \frac{W_{i,t}}{P_t} L_{i,t} + (1 + r_{t-1})\frac{B_{i,t-1}}{P_t} + r_t^K K_{i,t} + \frac{D_{i,t}}{P_t} + \\ -TAX_{i,t} - C_{i,t} - \frac{B_{i,t}}{P_t} - K_{i,t+1} + (1-\delta) K_{i,t} \end{array} \right] \end{array} \right\}$$

FOC wrt $C_{i,t}$: $\frac{1}{C_{i,t}} = \lambda_{i,t}$

FOC wrt $B_{i,t}$: $\frac{\lambda_{i,t}}{P_t} = \beta E_t \frac{\lambda_{i,t+1}}{P_{t+1}}(1 + r_t)$

FOC wrt $K_{i,t+1}$: $\lambda_{i,t} = \beta E_t \lambda_{i,t+1} \left[r_{t+1}^K + (1-\delta) \right]$

Model #1: A NK Model with Capital Accumulation
Households and Preferences

Combining the previous conditions we derive the Euler equation governing the time path of consumption

$$\frac{1}{C_{i,t}} = \beta E_t \frac{1}{C_{i,t+1}} \frac{1+r_t}{1+\pi_{t+1}}$$

and the asset equation according to which at the optimum a household is indifferent between the two assets (capital and risk-free public debt bonds) since the expected benefit in terms of utility is the same:

$$E_t \lambda_{i,t+1} \frac{1+r_t}{1+\pi_{t+1}} = E_t \lambda_{i,t+1} \left[r_{t+1}^K + (1-\delta) \right]$$

Model #1: A NK Model with Capital Accumulation
Wage Setting

There is a continuum of unions each of which represents workers of a certain type. Effective labour input hired by the intermediate-good firm j is a CES function of the quantities of the different labour types employed:

$$L_t(j) = \left(\int\limits_0^1 L_{i,t}(i)^{\frac{\sigma_L - 1}{\sigma_L}} di \right)^{\frac{\sigma_L}{\sigma_L - 1}}$$

where $\sigma_L > 1$ elasticity of substitution across different types of labour inputs. At the optimum (and under symmetry among firms) the demand for each variety of labour input i is

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\sigma_L} L_t$$

where $W_t = \left[\int_0^1 W_{i,t}^{1-\sigma_L} di \right]^{\frac{1}{1-\sigma_L}}$ such that $W_t L_t = \int\limits_0^1 W_{i,t} L_{i,t} di$.

Model #1: A NK Model with Capital Accumulation
Wage Setting

The union representing worker of type i will set $W_{i,t}$ in order to max the expected utility of household i given $L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\sigma_L} L_t$. The relevant part of the Lagrangian is

$$E_0 \beta^t \left[\frac{\omega}{1-v} \left(1 - L_{i,t}\right)^{1-v} + \lambda_{i,t} \frac{W_{i,t}}{P_t} L_{i,t} \right]$$

At the optimum:

$$\frac{W_{i,t}}{P_t} = \underbrace{\frac{\sigma_L}{\sigma_L - 1}}_{\text{wage markup}} \frac{\omega \left(1 - L_{i,t}\right)^{-v}}{\lambda_{i,t}}$$

Model #1: A NK Model with Capital Accumulation
The Government

The government budget constraint is

$$B_t = (1 + r_t)B_{t-1} + P_t G_t - P_t TAX_t$$

where $G_t = G \exp(\varepsilon_{g,t})$ and $\varepsilon_{g,t} = \rho_g \varepsilon_{g,t} + \xi_{g,t}$ with $\xi_{g,t} \sim iid.N(0, \sigma_g^2)$, while

$$P_t TAX_t = P_t \overline{TAX} + \tau P_t B_t$$

where τ is set in order to rule out any explosive path of the public debt ("passive" rule as meant by Leeper 1991).

Model #1: A NK Model with Capital Accumulation
The Central Bank

The monetary authority sets the short-term nominal interest rate in accordance with an interest rate rule

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\iota_r} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\iota_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\iota_y}\right]^{1-\iota_r} u_t^R.$$

where $R_t = 1 + r_t$, $\Pi_t = 1 + \pi_t$, $u_t^R = exp(\varepsilon_{u,t})$ and $\varepsilon_{u,t} = \rho_u \varepsilon_{u,t} + \xi_{u,t}$ with $\xi_{u,t} \sim iid.N(0, \sigma_u^2)$.

Model #1: A NK Model with Capital Accumulation
Equilibrium

Combining the above conditions, imposing symmetry between firms, households and unions the equilibrium of the economy is described by the following equations.

The Euler equation

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{1 + r_t}{1 + \pi_{t+1}}$$

The capital asset equation

$$\lambda_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^K + (1 - \delta) \right]$$

The Lagrange multiplier

$$\lambda_t = \frac{1}{C_t}$$

The wage equation

$$\frac{W_t}{P_t} = \frac{\sigma_L}{\sigma_L - 1} \frac{\omega (1 - L_t)^{-\nu}}{\lambda_t}$$

Model #1: A NK Model with Capital Accumulation
Equilibrium

The aggregate production function

$$Y_t = A_t L_t^\alpha K_t^{1-\alpha}$$

The demand of capital

$$r_t^k = (1 - \alpha) MC_t A_t L_t^\alpha K_t^{-\alpha}$$

The demand of labour

$$\frac{W_t}{P_t} = \alpha MC_t A_t L_t^{\alpha-1} K_t^{1-\alpha}$$

The inflation equation

$$1 - \gamma_p \left(\Pi_t - 1 \right) \Pi_t + \beta \gamma_p E_t \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\Pi_{t+1} - 1 \right) \frac{Y_{t+1}}{Y_t} \Pi_{t+1} = \left(1 - MC_t \right) \theta$$

Model #1: A NK Model with Capital Accumulation
Equilibrium

The budget constraint of the government

$$B_t = (1 + r_t)B_{t-1} + P_t G_t - P_t TAX_t$$

The tax rule

$$P_t TAX_t = P_t \overline{TAX} + \tau P_t B_t$$

The interest rate rule

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\iota_r} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\iota_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\iota_y}\right]^{1-\iota_r} u_t^R.$$

Model #1: A NK Model with Capital Accumulation
Equilibrium

The capital accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t$$

The resource constraint of the economy

$$Y_t = C_t + I_t + G_t + \frac{\gamma_p}{2} (\Pi_t - 1)^2 Y_t$$

The exogenous processes governing A_t, G_t and u_t^R

Model #1: A NK Model with Capital Accumulation
A remark on the inflation equation

Given the inflation equation

$$1 - \gamma_p \left(\Pi_t - 1 \right) \Pi_t + \beta \gamma_p E_t \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\Pi_{t+1} - 1 \right) \frac{Y_{t+1}}{Y_t} \Pi_{t+1} = \left(1 - MC_t \right) \theta$$

in a zero-inflation steady state $MC = \frac{\theta - 1}{\theta}$.

With **trend inflation** (and no indexation): $MC = \frac{\theta - 1}{\theta} + \gamma_p \frac{(1-\beta)(\Pi - 1)\Pi}{\theta}$.

The markup will be:

$$markup = \left(\frac{\theta - 1}{\theta} + \gamma_p \frac{(1-\beta)(\Pi - 1)\Pi}{\theta} \right)^{-1}$$

The markup is decreasing in the level of trend inflation. As a result: output (and so employment) is higher the higher Π. However, a higher fraction of output is eaten up by the adjustment costs.

Model #1: A NK Model with Capital Accumulation
Calibration

$\alpha = 2/3$	labour share
$\beta = 0.99$	discount factor
$\delta = 0.1/4$	depreciation rate
$v = 0.8$	preference parameter
$\gamma_p = 58.25$	degree of price rigidities $1 - \theta = 0.25$
$\theta = 6$	elasticity of subst. between goods
$\sigma_L = 5$	elasticity of subst. between labour inputs
$\rho_A = 0.9$	persistence of tech shock
$\rho_G = 0.9$	persistence of the public spending shock
$\rho_R = 0.9$	persistence of monetary policy shock

Model #1: A NK Model with Capital Accumulation
Calibration

$\Pi = 1$	inflation
$L = 0.3$	employment
$Y = 1$	output
$\iota_\pi = 1.5$	monetary policy parameter
$\iota_R = 0$	monetary policy parameter
$\iota_y = 0.5/4$	monetary policy parameter

Model #1: A NK Model with Capital Accumulation

Effects of a Technology Shock
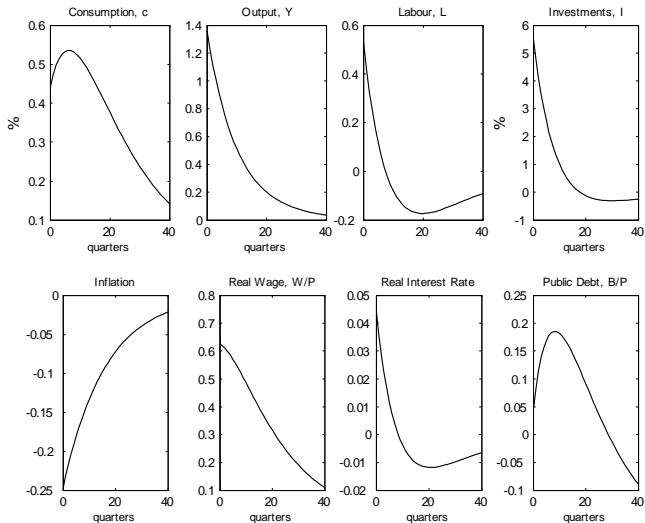
Model #1: A NK Model with Capital Accumulation
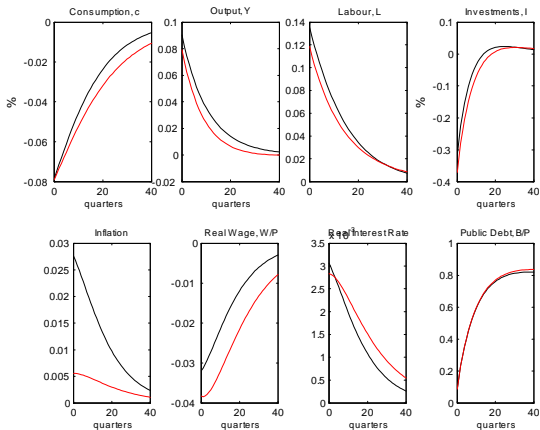Effects of a Technology Shock: Propagation

- Higher Productivity $\rightarrow$ lower marginal costs $\rightarrow$ lower inflation
- As in the basic RBC model a transitory productivity shock, which *temporarily* raises the real wage rate, increases employment (maybe we have a too low degree of price rigidity)
- Productivity $\uparrow$ $\rightarrow$MPK $\uparrow$ $\rightarrow$rental rate of capital $\uparrow$
    - Substitution effect increases savings (prevails)
    - Consumption increases gradually (*consumption smoothing*)
    - Investments increases on impact (*the volatile component*)
    - As a result *y* increases more than proportionally.

Model #1: A NK Model with Capital Accumulation

Effects of a Technology Shock with a higher degree of price rigidities[1]

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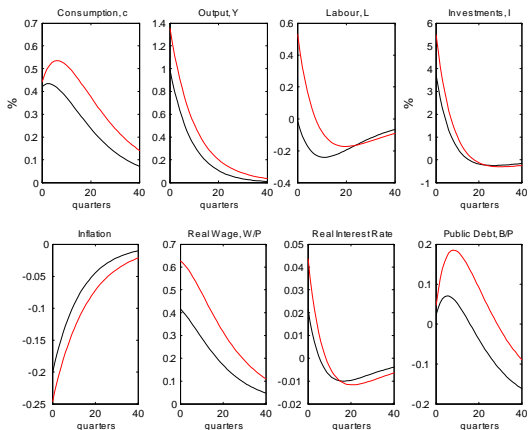
[1]Reponses under the baseline calibration are plotted in red.

Model #1: A NK Model with Capital Accumulation

Effects of a Government Spending Shock
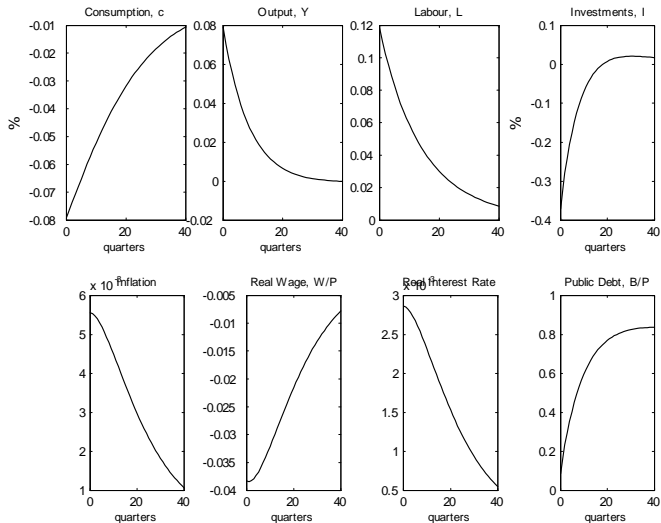
Model #1: A NK Model with Capital Accumulation
Effects of a Government Spending Shock: Propagation

- Government spending ↑ →taxes increase ↑ →net wage income↓
  - income effect (agents need to work more) →employment increases then gradually returns to normal
  - consumption falls, but the rise in government supply is temporary, hence agents respond by decreasing their capital holdings (consumption smoothing)
  - firms will revise their prices upward (as real marginal costs are higher)→ the Central Bank will react to inflation by increasing the nominal interest rate more than proportionally → as a result the real interest rate will increase (*lean against the wind policy*).
- As a result *y* will increase less than proportionally.
- Remark: no comovement between *c* and *g*.

Model #1: A NK Model with Capital Accumulation

Effects of a Government Spending Shock with a weak response to deviations from targets

$\iota_\pi = 1.1; \iota_y = 0$

Model #1: A NK Model with Capital Accumulation

Effects of a Monetary Policy Shock
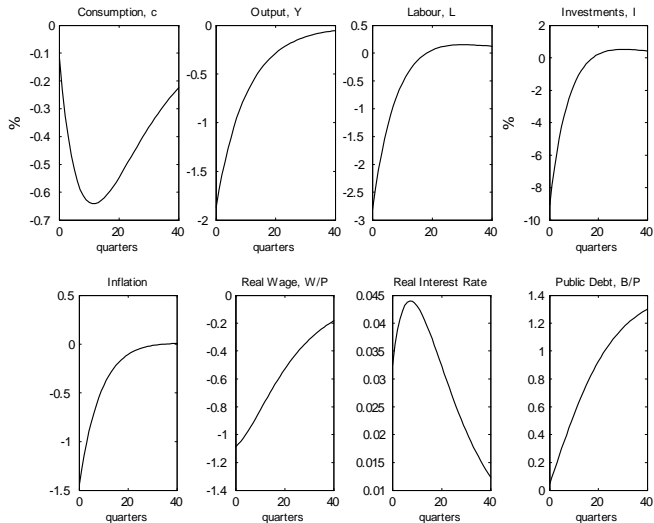
Model #1: A NK Model with Capital Accumulation
Effects of a Monetary Policy Shock: Propagation

The presence of price rigidities is a source of nontrivial real effects of monetary policy shocks.

Firms cannot immediatly adjust the price of their good when they receive new information about costs or demand conditions.

The shock generates an increase in the real rate, a decrease in inflation, output and employment.

Model #1: A NK Model with Capital Accumulation

Effects of a Monetary Policy Shock with a higher degree of price rigidities
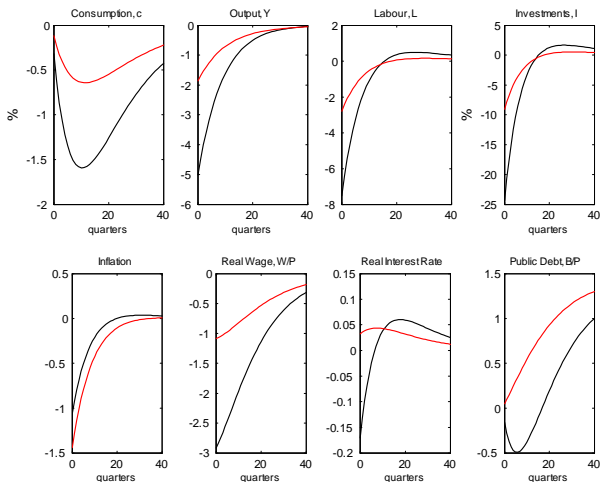
Model #2: Model #1+ Habit+Adjust. Costs

Extend the previous model to account for:

- external habit (see Lecture I)
- adjustment costs on investments (see Lecture I)
- adjustment costs on investments on labour (see Lecture I)

Model #2: Model #1+ Habit+Adjust. Costs
Households and Preferences

The typical household will solve the following problem
Household i is characterized by the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{i,t} - h^e \overline{C}_{t-1}) + \frac{\omega}{1-v} \left(1 - L_{i,t}\right)^{1-v} \right]$$

$\overline{C}$ average aggregate consumption; h^e measure of habit intensity
The period-by-period budget constraint is

$$
\begin{aligned}
P_t C_{i,t} + B_{i,t} + P_t I_{i,t} &= W_{i,t} L_{i,t} + (1 + r_{t-1}) B_{i,t-1} + P_t r_t^K K_{i,t} \\
&\quad + D_{i,t} - P_t TAX_t - P_t ADJ(I_t, K_t)
\end{aligned}
$$

where

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t}$$

$$ADJ(I_{i,t}, K_{i,t}) = \frac{\gamma_I}{2} \left(\frac{I_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t}$$

Model #2: Model #1+ Habit+Adjust. Costs
Households

At the optimum we now have (dropping index *i*)

$$\frac{1}{C_t - h_e \overline{C}_{t-1}} = \lambda_t$$

$$\frac{\lambda_t}{P_t} = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}} (1 + r_t)$$

$$\gamma_I \left(\frac{I_t}{K_t} - \delta_K \right) = q_t - 1$$

$$\underbrace{q_t \lambda_t}_{\xi_t} = \beta E_t \lambda_{t+1} r_{t+1} + \beta (1 - \delta) E_t \underbrace{q_{t+1} \lambda_{t+1}}_{\xi_{t+1}}$$

$$-\beta E_t \lambda_{t+1} \frac{\partial ADJ(I_{t+1}, K_{t+1})}{\partial K_{t+1}}$$

where q_t is the Tobin's marginal q.

Model #2: Model #1+ Habit+Adjust. Costs

Intermediate-goods firms the adjustment costs on labour

We now assume that hiring and firing unionized workers is costly, in particular we have:

$$ADJ\_L_t(j) = \frac{\gamma_L}{2} \left(\frac{L_t(j)}{L_{t-1}(j)} - 1 \right)^2 Y_t$$

As a result the optimal demand of labor is

$$
\begin{aligned}
\frac{W_t}{P_t} &= \alpha_L MC_t(J) \frac{Y_t(j)}{L_t(j)} - \gamma_L \left(\frac{L_t(j)}{L_{t-1}(j)} - 1 \right) \frac{Y_t}{L_{t-1}(j)} + \\
&+ \beta \gamma_L E_t \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{L_{t+1}(j)}{L_t(j)} - 1 \right) \frac{L_{t+1}(j)}{L_t(j)^2} Y_{t+1}
\end{aligned}
$$

Model #2: Model #1+ Habit+Adjust. Costs
The resource constrain of the economy

The resource constraint of the economy is now

$$
\begin{aligned}
Y_t \;=\; & C_t + I_t + G_t + \\
& + \frac{\gamma_p}{2} \left(\Pi_t - 1 \right)^2 Y_t + \\
& + \frac{\gamma_L}{2} \left(\frac{L_t \left(j \right)}{L_{t-1} \left(j \right)} - 1 \right)^2 Y_t + \\
& + \frac{\gamma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t
\end{aligned}
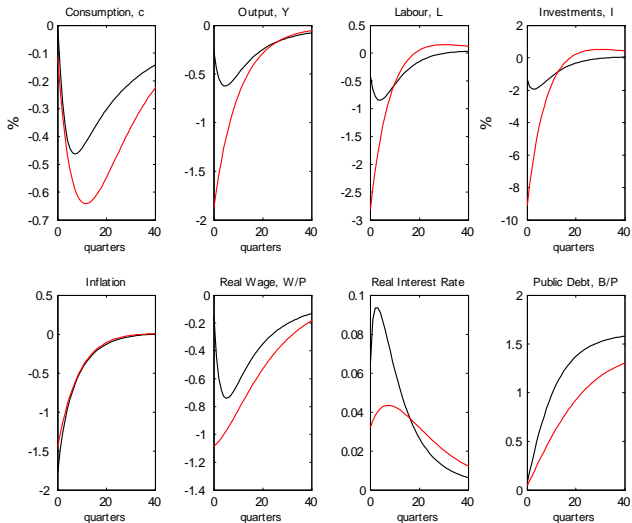$$

Model #2: Model #1+ Habit+Adjust. Costs

Effects of a Technology Shock
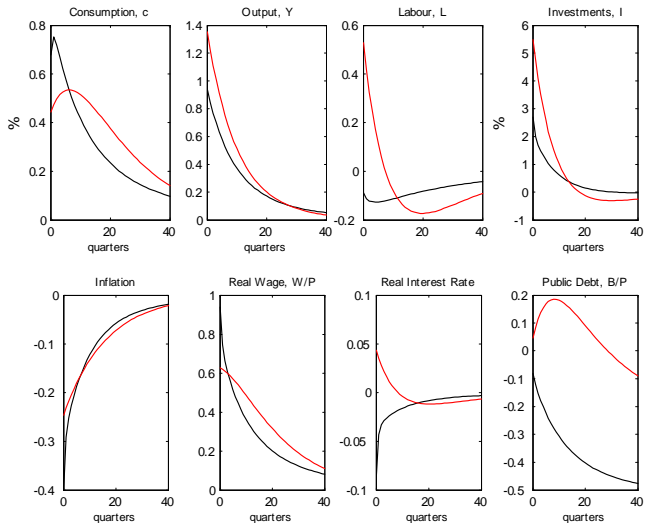
Model #2: Model #1+ Habit+Adjust. Costs

Effects of a Government Spending Shock
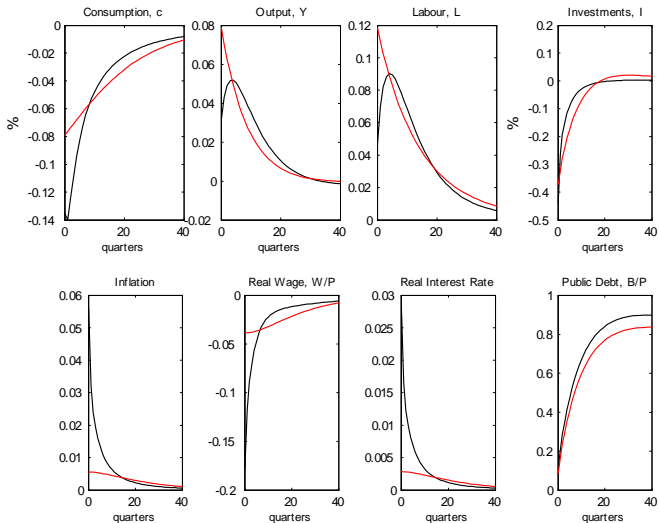
Model #2: Model #1+ Habit+Adjust. Costs

Effects of a Monetary Policy Shock

Model #3: Model #1+RoT

Extend the Model #1 to account for:

- Consider rule of thumb households as in Galí, López-Salido and Vallés (2007).

Model #3: Model #1+RoT
Households

There is a continuum of households. Population is constant and normalized to 1.

A fraction s_{NR} of households do not borrow and save, and just consume their current labor income (hand-to-mouth households) → extreme form of non-Ricardian behavior

Motivation: an extensive empirical literature provides evidence of "excessive" dependence of consumption on current income; deviations from the permanent income hypothesis.

As a result now we have two types of households:

- Non-Ricardian households (population share s_{NR})
- Ricardian households (population share $1 - s_{NR}$)

Model #3: Model #1+RoT
The Ricardian Households

The typical Ricardian household will solve the following problem
Household *i* is characterized by the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{i,t}^R - h^e \overline{C}_{t-1}) + \frac{\omega}{1-v} \left(1 - L_{i,t}^R\right)^{1-v} \right]$$

$\overline{C}$ average aggregate consumption; h^e measure of habit intensity
The period-by-period budget constraint is

$$
\begin{aligned}
P_t C_{i,t}^R + B_{i,t}^r + P_t I_{i,t}^r &= W_{i,t} L_{i,t}^R + (1 + r_{t-1}) B_{i,t-1}^R + \\
&\quad + P_t r_t^K K_{i,t}^R + D_{i,t}^R - P_t TAX_t^R
\end{aligned}
$$

where

$$K_{i,t+1}^R = (1 - \delta) K_{i,t}^R + I_{i,t}^R$$

Model #3: Model #1+RoT
The Ricardian Households

At the optimum we now have (dropping index *i*) we have the standard optimality conditions:

$$\frac{1}{C_t^R} = \lambda_t^R$$

$$\frac{\lambda_t^R}{P_t} = \beta E_t \frac{\lambda_{t+1}^R}{P_{t+1}}(1 + r_t)$$

$$\lambda_{i,t} = \beta E_t \lambda_{i,t+1} \left[r_{t+1}^K + (1 - \delta) \right]$$

Model #3: Model #1+RoT
The Non-Ricardian Households

Non-Ricardian households are assumed to behave in a "hand-to-mouth" fashion: they fully consume their current labor income (no consumption smoothing).

The representative household of this category derives utility from consumption and leisure:

$$\log C_{i,t}^{NR} - \frac{\omega}{1-v}\left(1 - L_{i,t}^{NR}\right)^{1-v_{N_A}}$$

given a flow budget constraint of the form:

$$P_t C_{i,t}^{NR} = W_{i,t} L_{i,t}^{NR} - P_t TAX_{i,t}^{NR}$$

Consumption function is then:

$$C_{i,t}^{NR} = \frac{W_{i,t}}{P_t} L_{i,t}^{NR} - TAX_{i,t}^{NR}$$

Model #3: Model #1+RoT
Aggregate variables

Aggregate consumption is now defined as

$$C_t \equiv s_{NR} C_t^{NR} + (1 - s_{NR}) C_t^R$$

while investments and capital aggregates of the economy are give by

$$I_t \equiv (1 - s_{NR}) I_t^R$$

$$K_t \equiv (1 - s_{NR}) K_t^R$$

Model #3: Model #1+RoT
Wage Setting

New assumption: The fraction of Non-Ricardian and Ricardian households is uniformly distributed across workers types and hence across unions. Each period a typical union representing worker i sets the wage for its workers in order to maximize the objective function of the form

$$s_{NR} \left[\frac{\omega}{1-v} (1 - L_{i,t})^{1-v} + \lambda_{i,t}^{NR} \frac{W_{i,t}}{P_t} L_{i,t} \right] +$$
$$+ (1 - s_{NR}) \left[\frac{\omega}{1-v} (1 - L_{i,t})^{1-v} + \lambda_{i,t}^{R} \frac{W_{i,t}}{P_t} L_{i,t} \right]$$

s.t. to the demand schedule: $L_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\sigma_L} L_t$. At the optimum the wage equation is

$$\frac{\sigma_L}{\sigma_L - 1} \omega (1 - L_t)^{-v} = \left[s_{NR} \lambda_t^{NR} + (1 - s_{NR}) \lambda_t^R \right] \frac{W_t}{P_t}$$

Model #3: Model #1+RoT
The government

The government budget constraint is

$$B_t = (1 + r_t)B_{t-1} + P_t G_t - P_t TAX_t$$

where $G_t = G \exp(\varepsilon_{g,t})$ and $\varepsilon_{g,t} = \rho_g \varepsilon_{g,t} + \xi_{g,t}$ with $\xi_{g,t} \sim iid. N(0, \sigma_g^2)$, while

$$TAX_t \equiv s_{NR} TAX_t^{NR} + (1 - s_{NR}) TAX_t^R$$

where we assume that $TAX_t^{NR} = TAX_t^R = TAX_t$
The fiscal rule is as before:
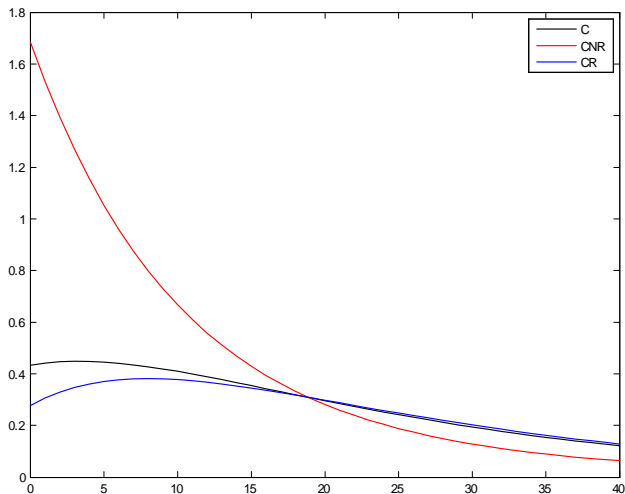
$$P_t TAX_t = P_t \overline{TAX} + \tau P_t B_t$$

Set $s_{NR} = 0.20$.

Model #3: Model #1+ RoT

Effects of a Technology Shock
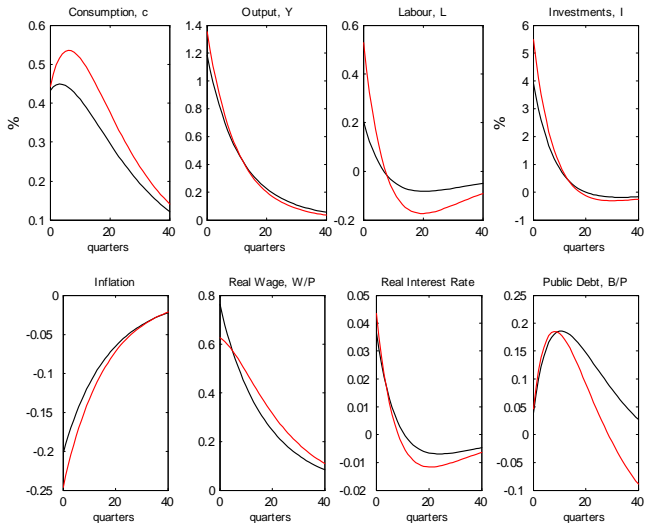
Model #3: Model #1+ RoT

Effects of a Technology Shock: Consumption paths

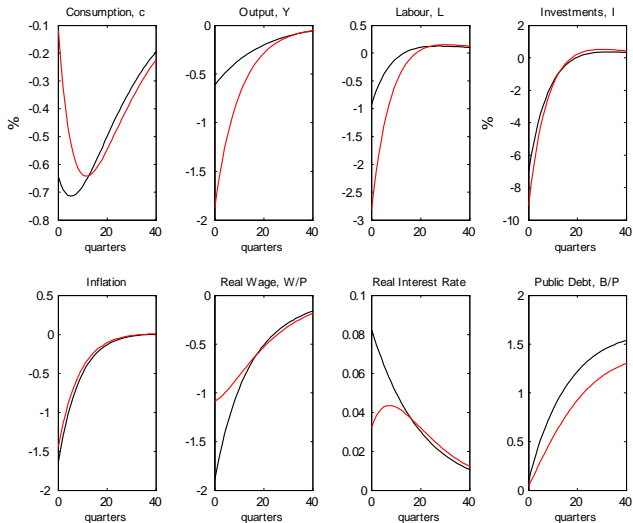# Model #3: Model #1+ RoT

Effects of a Government Spending Shock
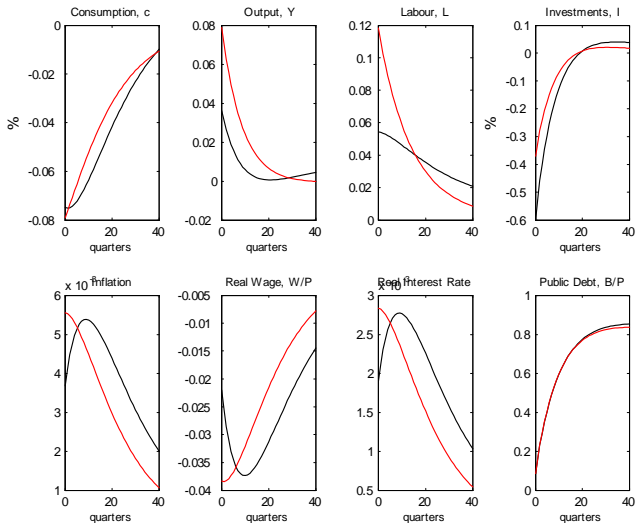
Model #3: Model #1+ RoT

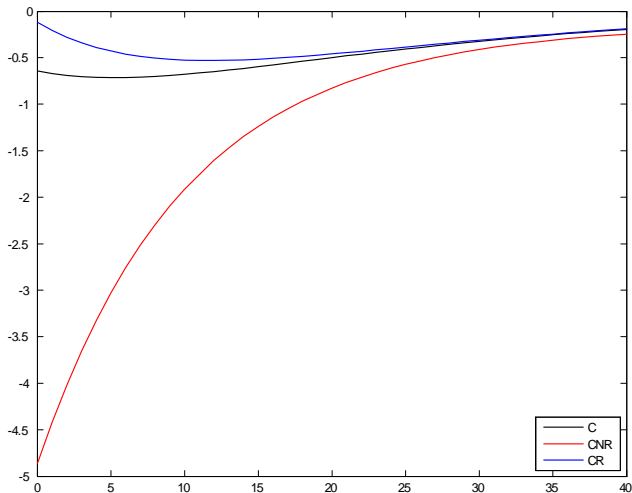Effects of a Government Spending Shock: Consumption paths
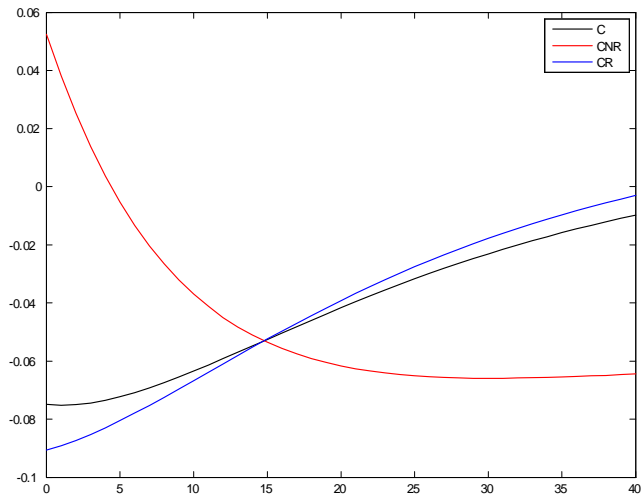
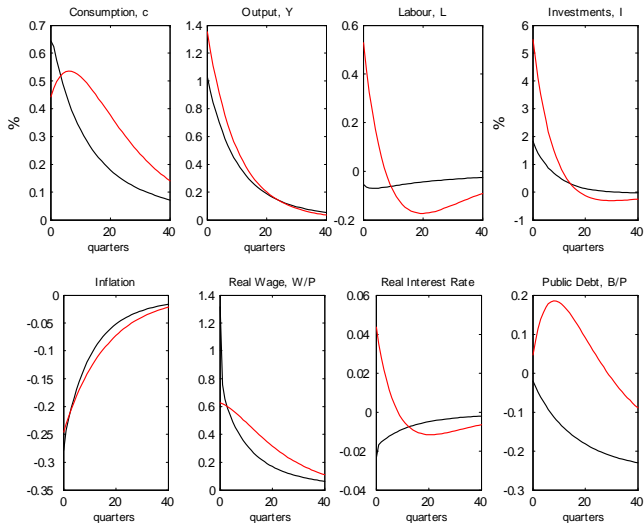# Model #3: Model #1+ RoT

Effects of a Monetary Policy Shock

Model #3: Model #1+ RoT
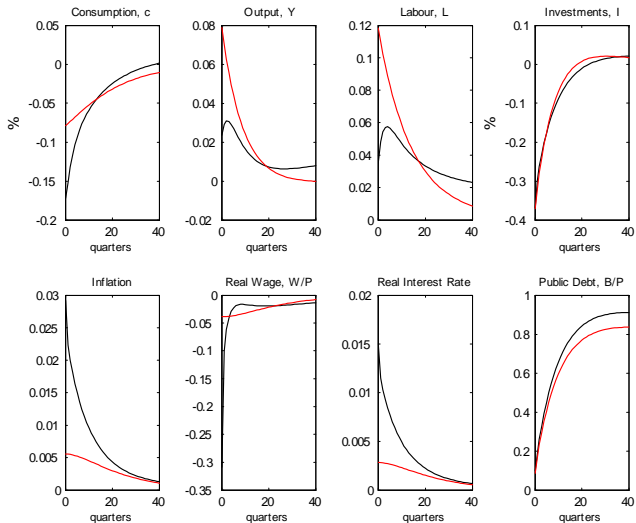
Effects of a Monetary Policy Shock: Consumption Paths

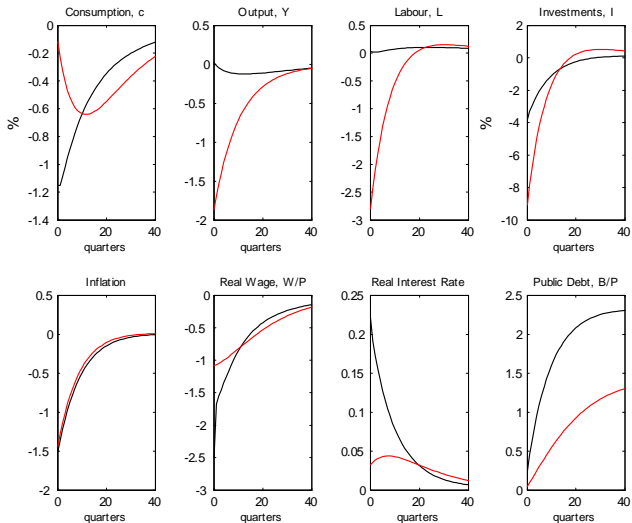# Model #4: Habit+Adjust. Costs+RoT

Effects of a Technology Shock

Model #4: Habit+Adjust. Costs+RoT

Effects of a Government Spending Shock

Model #4: Habit+Adjust. Costs+RoT

Effects of a Monetary Policy Shock

Discussion
What is still missing to have a more complete model?

- Indexation
- Wage rigidities
- Variable capacity utilization
- International trade and international capital markets

Discussion
Challenges of DSGE modelling

- Labour migration and remittances
- Foreign direct investments
- Heterogenous workers (atypical, self-employed etc...)
- Informal sector
- Role of relative price movements
- Non-market sector (public goods)
- Financial market frictions
- Portfolio choice
- Term structure of interest rates
- Currency risk premia
- Endogenous growth
- Time varying parameters and structural breaks
- Estimation problems

References

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