

Department of the Treasury





N° 16 - May 2012

ISSN 1972-411X

Uncertainty and Heterogeneity in Factor Models Forecasting

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Working Papers

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Uncertainty and Heterogeneity in Factor Models Forecasting

Matteo Luciani (*), Libero Monteforte (**)

Abstract

In this paper we propose to exploit the heterogeneity of forecasts produced by different model specifications to measure forecast uncertainty. Our approach is simple and intuitive.

It consists in selecting all the models that outperform some benchmark model, and then to construct an empirical distribution of the forecasts produced by these models. We interpret this distribution as a measure of uncertainty. We perform a pseudo real-time forecasting exercise on a large database of Italian data from 1982 to 2009, showing case studies of our measure of uncertainty.

JEL classification: C13, C32, C33, C52, C53.

Keywords: Factor Models, Model Uncertainty, Forecast Combination, Density Forecast.

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We would like to thank Cecilia Frale for helpful comments. An earlier version of this paper was written while the authors were at the Italy's Ministry of the Economy and Finance, Treasury Department. Luciani gratefully acknowledged financial support from the Belgian National Bank and the IAP P6/07 contract, from the IAP program (Belgian Scientific Policy), "Economic policy and finance in the global economy". Luciani is postdoctoral researcher of the F.R.S.-FNRS and gratefully acknowledges their financial support.



1 Introduction

Since the seminal papers of Stock and Watson (2002a) and Forni et al. (2005) factor models are increasingly used for macroeconomic forecasting by central banks, governments, and market operators. The good performance of the model¹ has stimulated further research, and the literature has suggested many refinement and improvements (Bai and Ng, 2008, 2009).

Nowadays, there exists a large number of ways to produce a forecast with a factor model. There are different type of models (dynamic vs. static); different estimation methods (principal components, LARS, Boosting); and, finally, each of these models can be specified in many different ways by simply changing, for example, the number of factors, or the number of lags.

Although theoretically equally acceptable, these different factor models might end-up by producing very different forecasts. However, this heterogeneity is a big problem in real-time forecasting. The standard procedure is to select the best model, i.e. the type, the estimation method, and the model specification that minimizes some criterion, and then to discard the remaining models. We believe, however, that this practice is restrictive and that it does not exploit all available information as, for example, considering alternative scenarios.

In this paper we propose an approach for forecasting with factor models that is able to exploit the heterogeneity of forecasts, and that interprets this heterogeneity as a special category of model uncertainty. This approach is relevant for policy making since, by exploiting the forecasts of those models whose performance is very similar to the one of the best model, it provides a warning of additional possible scenarios.

Our method is extremely intuitive. It consists in selecting all the models that outperform some benchmark model, and then in constructing approximations of the empirical distribution of all the forecasts produced by these models. We interpret this distribution as a measure of uncertainty. By means of a pseudo real-time forecasting exercise on Italian data, we show that albeit surprisingly simple, our method is meaningful and effective.

Our approach is related with two strands of the literature. On the one hand, we share the idea of using (many) different models with the forecast combination literature (Bates and Granger, 1969; Timmermann, 2006). On the other hand, we share the aim of assessing uncertainty with the density forecast literature (Diebold et al., 1998; Tay and Wallis, 2000). However, differently from the former, we suggest to exploit a large number of models to measure forecast uncertainty rather than for reducing the prediction error. Differently from the latter, we assess the uncertainty between models rather than uncertainty within a model (i.e. the stochastic variability of coefficients and shocks for a given model). This kind of uncertainty is particularly relevant from a policy perspective since it shows how at the same point in time, and with the same information set, different researchers (or institutions) may produce different forecasts. This approach is new in the literature and produces distributions

¹See among others Stock and Watson (2002b), Forni et al. (2003), Boivin and Ng (2005), Artis et al. (2005), Schumacher (2007, 2010), D'Agostino and Giannone (2011), and, for a review, Eickmeier and Ziegler (2008).



of the forecast that are characterized by being not "well behaved". Our forecast distributions are often bimodal, asymmetric and with tails not necessarily increasing with the forecast horizon.

The paper is organized as follows. Section 2 describes the methodology, while section 3 explains how we constructed a large number of models. Section 4 presents the empirical application by first comparing all the estimated models, and by then explaining how to interpret the different forecasts as a measure of uncertainty. Section 5 concludes.

2 Methodology

In this section, we review the methodologies that we will use to estimate our factor models. Results are not derived rather simply illustrated, and, therefore, we refer the reader to the papers of Stock and Watson (2002a), Efron et al. (2004), Forni et al. (2005), Bai and Ng (2008), and Bai and Ng (2009) for technical details and proofs.

Throughout this section we will refer to the variable for which we want to make a prediction h step ahead as y_{t+h}^h , and we will refer to the N potential predictors as x_t .

2.1 Diffusion Indexes

Let x_t be an $N \times 1$ vector of zero mean stationary variables that admits a *static* factor representation such as:

$$x_t = \Lambda F_t + \xi_t = \chi_t + \xi_t, \quad \text{for} \quad t = 1, \dots, T, \tag{1}$$

where F_t is an $r \times 1$ vector containing the static factors, Λ is an $N \times r$ matrix of factor loadings, and χ_t and ξ_t are $N \times 1$ vectors containing respectively the common and the idiosyncratic component. The *Diffusion Index* proposed by Stock and Watson (2002a) consists in forecasting y_{t+h}^h by augmenting an autoregressive model with the first r factors and their first p_f lags:

$$y_{t+h}^{h} = \alpha(L)y_t + \beta(L)F_t + \varepsilon_t \tag{2}$$

where $\alpha(L)$ and $\beta(L)$ are polynomials of order p_y and p_f respectively. Stock and Watson (2002a) demonstrates that, if the idiosyncratic components ξ_t are mildly serial and cross-sectional correlated, the static factors in (1) can be consistently estimated with the method of principal components. Having estimated the static factors, Stock and Watson (2002a) suggest to estimate equation (2) via OLS. Consistency of such procedure is proved in Bai and Ng (2006).



2.2 Dynamic Factor Models

Let x_t be an $N \times 1$ vector of zero mean stationary variables that follows a "Dynamic Factor Model" such as:

$$x_t = C(L)\eta_t + \xi_t = \chi_t + \xi_t, \text{ for } t = 1, ..., T$$
 (3)

where η_t is a $q \times 1$ vector of dynamic factors, with $q \ll N$, and $C(L) = \sum_{j=0}^{\infty} C_j L^j$ is an $N \times q$ matrix polynomial in the lag operator with square summable entries. Let us suppose that y_t is one of the entries of a vector x_t , say the *i*-th entry for simplicity, then a forecast of $y_{t+h}^h \equiv x_{i,t+h}^h$ can be obtained as the sum of the forecast of the common component and of the idiosyncratic component: $x_{i,t+h}^h = \chi_{i,t+h}^h + \xi_{i,t+h}^h$. Forni et al. (2005) (proposition 4) demonstrate that by means of a two-step estimator a forecast of the common component that converges to the best linear forecast of $\chi_{i,t+h|t}$ can be obtained, while they suggest that the idiosyncratic component can be neglected.²

Step 1: Let $\tilde{\Sigma}^{\chi}(\theta)$ and $\tilde{\Sigma}^{\xi}(\theta)$ be the estimated spectral density matrix of, respectively, the common and the idiosyncratic component obtained with the method of dynamic principal components, then the covariance matrices of χ_t , $\tilde{\Gamma}_k^{\chi}$, and ξ_t , $\tilde{\Gamma}^{\xi}(\theta)$, can be consistently estimated as the inverse Fourier transform of, respectively, $\tilde{\Sigma}^{\chi}(\theta)$ and $\tilde{\Sigma}^{\xi}(\theta)$.

Step 2: Let \widehat{Z} be the $N \times r$ matrix containing the first normalized r eigenvectors of $\widetilde{\Gamma}_0^{\chi}(\widetilde{\Gamma}_0^{\xi})^{-1}$, then the static factors can be estimated as the first r generalized principal components of x_t , $\widehat{F}_t = \widehat{Z}'x_t$. The factor loadings Λ can then be recovered as the linear projection of the static factors on x_t , $\Lambda = \widetilde{\Gamma}_0^{\chi} \hat{Z}(\hat{Z}'\widetilde{\Gamma}_0^{\chi}\hat{Z})^{-1}$. Having estimated both the factors and the loadings the forecast of the common components is obtained as: $\widehat{\chi}_{t+h|t} = \widetilde{\Gamma}_h^{\chi} \hat{Z}(\hat{Z}'\widetilde{\Gamma}_0^{\chi}\hat{Z})^{-1}\hat{Z}'x_t$.

2.3 Least Angle Regressions (LARS)

The idea of least angle regression is to build recursively an estimate of y by $x\hat{\beta}$ where at each stage a regressor is added. At the first stage the variable mostly correlated with y, say x_j , is selected, and an OLS regression of y on x_j is run. Define the residual of the first step as $v = y - \gamma \hat{\beta}_j x$, where γ is the step length, then the algorithm take the largest step towards the direction of this predictor until it finds another regressor, say x_l , as much correlated with v. Then, the LARS algorithm searches for the third variable equiangularly between x_j and x_l . At the k-th step, $\hat{\beta}$ has k non zero elements, and N-k zero elements. In this way the variables mostly correlated with y are included one at a time, but, at the same time, LARS avoids selecting variables that are too "similar". One of the main features of LARS is that the direction of the search, and the updating rule are computed endogenously by the algorithm;

²A refinement of the Forni et al. (2005) procedure is proposed in D'Agostino and Giannone (2011) which suggest to forecast the idiosyncratic component as the linear projection of $\xi_{i,t+h|t}$ on $[x_{i,t} \ x_{i,t-1} \ \dots \ x_{i,t-p}]$.



the researcher needs simply to set the number of iterations.

2.4 Boosting

Let $z_t = \{x_{1,t}, \dots, x_{1,t-p_x}, \dots, x_{N,t}, \dots, x_{N,t-p_x}\}'$, be the $\bar{N} \times 1$ matrix containing all the N variables and their p_x lags, the idea of Boosting is to build an estimate of y_{t+h} by recursively estimating regressions of y_{t+h} on z_{jt} , where z_{jt} is the variable more powerful in predicting y_{t+h} . At each step the prediction is updated by $\hat{\mu} = \gamma \hat{b}_j z_{jt}$, where γ is the step length, and $\hat{b}_j z_{jt}$ is the linear projection of z_{jt} on u_{t+h} , the residual obtained in the previous step. At the k-th iteration, the estimator $\hat{\beta}_k$ is obtained as $\hat{\beta}_k = \hat{\beta}_{k-1} + \gamma \hat{b}_k^{\dagger}$, where \hat{b}_k^{\dagger} is an $\bar{N} \times 1$ vector where all entries are zero but element j, and the forecast of y_{t+h} is obtained as $\hat{y}_{t+1}^k = \hat{\beta}_k z_t$.

Bai and Ng (2009) suggest two algorithms to perform boosting, the first called *component-wise* algorithm treats each entry in Z_t as a distinct variable, while the *block-wise* algorithm treats lags of the same variable jointly.

3 Constructing a large number of factor models

In this section we explain how we constructed a large number of forecasts. Forecasts are produced by means of eight different methods that can be grouped in two main types: Diffusion Indexes, and Dynamic Factor models. Table 1 presents the complete list of methods used in this papers.

Table 1: Estimated Models

Ν°	Model	Forecast Equation	0	AR
0	AR	$y_{t+h}^h = a(L)y_t + v_t$		${\it Diffusion\ Indexes:}$
Diff	usion Inde	xes:		
Mo	M .1 1	В 1 Б 1:	Factors	Estimation
Ν°	Method	Forecast Equation	Extracted from:	Method
1	DI	$y_{t+h}^{h} = \alpha(L)y_t + \beta(L)F_t + \varepsilon_t$	x_t	OLS
2	DI2	ST ST	$[x_t \ x_t^2]$	OLS
3	LDI	en en	$ ilde{x}_t$	OLS
4	DIB	. 	x_t	Boosting
5	DI2B		$[x_t x_t^2]$	Boosting
Dyn	amic Fact	or Models:		
N°	Method	Forecast Equation	Estimation of	
			idiosyncratic component	
6	FHLR_a	$x_{t+h}^h = \chi_{t+h}^h$	none	

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Method DI is the classical diffusion index proposed by Stock and Watson (2002a), while methods DI2, LDI, DIB, and DIB2 are all variant of DI. Originally proposed by Bai and Ng (2008), DI2 consists in extracting the factors from a panel including both the normal variables and their squared values, and then in estimating a diffusion index. Similarly, LDI (Bai and Ng, 2008) consists in extracting the factors from a panel including only few predictors selected with the LARS algorithm, and then in estimating a diffusion index. Finally, DIB and DIB2 (Bai and Ng, 2009) consists in estimating equation (2) by Boosting rather than by OLS.

Method DF_a simply implements the proposal of Forni et al. (2003, 2005), while DF_b implements the refinement suggested by D'Agostino and Giannone (2011).

As said in the introduction, within each of these methods we can produce different forecasts simply by choosing different model specifications, i.e. by varying the number of static/dynamic factors, or the number of lags. Above all, a priori all these methods and specifications are (theoretically) equally acceptable. In this paper we have 267 different factor forecast, plus 4 different benchmark AR forecasts. Table 2 presents the complete list of specifications used in this papers.

Table 2: List of Model Specifications

 AR: we performed forecasts for p = 1,,4, where p is the order of the autoregression fications); DI: we allow p_y = 1,,4, p_f = 1,,4, r = 1,,5, where p_y and p_f are the number of, respectively, the endogenous variable and the static factors, and r is the number factors (80 specifications); DI2: same as DI (80 specifications); 	4 speci-
DI: we allow $p_y = 1,, 4$, $p_f = 1,, 4$, $r = 1,, 5$, where p_y and p_f are the number of, respectively, the endogenous variable and the static factors, and r is the number factors (80 specifications); DI2: same as DI (80 specifications);	
of, respectively, the endogenous variable and the static factors, and r is the number factors (80 specifications); DI2: same as DI (80 specifications);	
of, respectively, the endogenous variable and the static factors, and r is the number factors (80 specifications); DI2: same as DI (80 specifications);	of lags
DI2: same as DI (80 specifications);	
LDI: same as DI, but the matrix \tilde{x}_t from which factors are extracted contains half of the v	ariables
in x_t , meaning the first 59 variables selected by the LARS algorithm (80 specification)	ons);
DIB: we include all possible regressors in the forecast equation $(p_y = 4, p_f = 4, \text{ and})$	
24 regressors), we set the step length γ equal to 0.5, we estimate the model by h	oth the
component-wise and the block-wise algorithm, and we save the forecast obtained af	er 5, 10
and 20 iterations (6 specifications);	
DIB2: same as DIB (6 specifications);	
DF _a : we select 3 dynamic factors as indicated by both the Hallin and Liška (2007) crite	ria, and
the Onatski (2009) test, and we allow for a number of static factors that varies be	tween 3
and 5, a range consistent with the indication obtained from information criteria (Bai and
Ng, 2002; Alessi et al., 2010) (3 specifications);*	
DF_b : for the common component same as DF_a , while for the idiosyncratic component we p	roduced
forecasts by setting $p_{\xi} = 1, \dots, 4$ (12 specifications).	

^{*} To save space results of these tests are not reported here.

The factors are extracted from a panel of 118 quarterly series, 100 describing the Italian economy, and 18 representing the rest of the world. The variables cover different categories: GDP and Components, Value Added by Sector, Unit labor cost, Employee Compensation, Employment, Interests Rates, Monetary Aggregates, Prices, Industrial Production, Exchange Rates, Business, and Confidence and Survey indicators. In addition, to account for world business cycle fluctuations we also include GDP, CPI, and the Unemployment Rate for France,



Germany, UK, US and Japan, and the Interest Rate of UK, US and Japan. All variables are first transformed to reach stationarity and then demeaned and standardized. As in Stock and Watson (2002b) we take the second difference of the logarithm of both prices and monetary indicators, and the first difference of interest rates. After transformation, all variables are stationary according to the Augmented Dickey Fuller test. For any further information on the database, the complete list of variables and transformations is reported in the Appendix.

We use the method of direct forecast (Stock and Watson, 2002b): let Y_t be the raw variable assumed to be integrated of order one, then y_{t+h}^h is defined as: $y_{t+h}^h = \log(Y_{t+h}) - \log(Y_t)$, that is the growth rate between period t and period t+h. On the other hand, the autoregressive variable on the right-hand side y_t is defined as $y_t = \log(Y_t) - \log(Y_{t-1})$.

4 Empirical Analysis

4.1 Comparing Factor Based Forecasts

In this section we evaluate the performance of different factor models. Forecasts are produced by means of a recursive scheme, and are computed with a forecast horizon from 1 to 8 steps ahead. The first estimation is carried out on a sample from 1982:3 to 2002:2 (T=80), while the last estimation on a sample from 1982:3 to 2009:2. Overall we produced 29 forecasts for the one step ahead, 28 for the two steps ahead, and 22 for the 8 steps ahead.

From table 3 to table 7 we present relative mean squared errors for a large number of macroeconomic variables. The benchmark is an AR forecast. An entry lower than 1 means that the m-th model beats the benchmark AR forecast, while an entry greater than 1 means that model m does worse than an AR. For each method we select the best specification, meaning the one that, within the range of different parameters configurations presented in section 3, produces the smaller mean squared error.

In this section, we provide a simple bird eye view of the results by variable. The goal is to identify for which variable factor models can improve with respect to a simple AR model:

GDP: Factor models outperform the AR model when forecasting GDP both in the short run and in the long run (table 3). Noteworthy, the gain from factor based forecasts is increasing at longer time horizons.

Labor Market: Factor models do quite well when predicting the number of persons employed, while they are outperformed by the AR model when predicting the unemployment rate (table 4). With respect to employment in different sectors, factor forecasts

³Given that the outcome of models 6-7 is different from the one obtained with models 1-5, some manipulations are needed for correct comparison. Let X_{it} be the non standardized growth rate of the *i*-th variable, then $x_{it} = (X_{it} - \mu_{X_i})/\sigma_{X_i}$, and therefore $x_{i,t+h}^h = (X_{i,t+h}^h - \mu_{X_i})/\sigma_{X_i}$. Hence, when forecasting with DF_a and DF_b we have that Y_{t+h}^h can be obtained as $Y_{t+h}^h = \sum_{j=1}^h (x_{i,t+j}^j \sigma_{X_i} + \mu_{X_i})$.



deliver good results for industrial employment. Whereas, when forecasting other sectors the advantage of a large information set is negligible.

Gross Value Added: Factor models perform particularly well when predicting VA in the services sector (table 5). They also perform well when predicting education, health, and other private & public services.

Consumption: Factor models consistently improve with respect to the AR model when predicting aggregate consumption (table 6). In particular, their performance is good when predicting non durables goods consumption and services consumption.

Investments: Factor models do better than the AR benchmark at the one step ahead horizon, while their performance is similar at longer forecast horizons (table 7).

Summing up, in line with the applied literature (Boivin and Ng, 2005; D'Agostino and Giannone, 2011; Schumacher, 2007) our results show that factor models outperform autoregressive models in forecasting most macroeconomic variables. Moreover, albeit some exception, we find that Diffusion Index type forecasts tend to do better than Dynamic Factor Model type forecasts.⁴

To conclude this section, in table 8 we report the number of specifications within each type of factor models that does worse than the benchmark AR when predicting GDP. Results show that most specifications (i.e. no matter the number of factors, the number of lags, etc.) perform better than the AR. Moreover, none of the estimated factor models perform worse than the AR after the 4^{th} forecast horizon. These result justify our approach: since most of the 267 estimated models have, at least some, predictive power, why selecting only one of them?

4.2 Two Examples of Model Uncertainty

As we have just shown, there exists a large number of ways to produce a macroeconomic forecast with a factor model. There are different type of models, different estimation methods, and each of these models can be specified in many different ways. However, although (i) theoretically all these models are equally acceptable, and (ii) most of them outperform a standard AR model (table 8), they might end-up by producing very different forecasts. The question then is: can we somehow exploit this different outcomes?

⁴There exists a wide applied literature that has compared the forecasting performances of DI vs. FHLR type forecasts without, however, reaching a conclusion. den Reijer (2005), Cheung and Demers (2007), and Schumacher (2007), compare DI vs FHLR when forecasting GDP for respectively the Netherlands, Canada, and Germany. den Reijer (2005), and Schumacher (2007) find that FHLR outperforms DI, while Cheung and Demers (2007) find no noticeable differences between the two methods. Boivin and Ng (2005) by analyzing US monthly data on a large number of series, conclude that DI performs better because it does not impose a factor structure and thus the forecast can more easily adapt to the data. D'Agostino and Giannone (2011), by analyzing a similar dataset, criticized this conclusion and find that FHLR does similar compared to DI. For a complete review of factor model forecasting performance see Eickmeier and Ziegler (2008).



Indeed, the literature has already addressed this issue, and it is now well known that by combining different forecast the prediction error may reduce (Timmermann, 2006). However, what we claim here is that the different forecasts can be used to measure forecast uncertainty in the context of factor modeling.

In the following, we explain how our method works. This method has the desirable feature of being extremely intuitive, since it consists in selecting all the models that outperform some benchmark model, and then in approximating the empirical distribution of the forecasts produced by all these models. We interpret this distribution as a measure of uncertainty. Since our approach is mainly aimed at policy maker, we present it here by making use of a practical example.

Suppose that in the middle of the global crisis, say beginning of 2009, we were asked by the policy maker to provide forecasts for the next two years. To mimic this situation, we produce pseudo real time forecasts of GDP with our 267 factor models. Then the question is: what is the relevant information that we want to report to the policy maker?

The first option is to use the standard approach: we identify the *best* model for each forecast horizon, and then we report the implied path of forecasts. Table 9 presents pseudo real time forecast of GDP for 2009 and 2010. Each entry reports the predicted average percentage quarter-on-quarter growth rate between t and t+h: $100 \times \frac{1}{h}(\widehat{GDP}_{t+h}-GDP_t)$. If we reported only the path of forecast suggested by the *best* models (bold entries), we would have depicted to the policy maker a critical situation (i.e. negative growth rates for the following two years). However, except for this statement, we would have not been able to say much more. We could have told what the *best* forecast is, but we could have not considered alternative scenarios delivered by equally acceptable models. Our method is aimed at this.

In figure 1 we show the pseudo real time forecast of the 20 best models in terms of mean squared error, i.e. the 20 factor models (independently from their type/estimation method/specification) that produce the smaller MSE. As we can see, although we are considering models with similar predicting ability, the forecast that they produce is very different thus showing a high degree of uncertainty. Moreover, the best forecast (black bar) is among the most, say, pessimistic models. However, despite this additional piece of information, the main conclusion of our report would have not changed since 18 out of 20 models predicted negative growth.

The question then is: why restricting the analysis to twenty models? What happen if we consider a higher number of models? In figure 2 we answers this question.

Figure 2 shows the distribution of the 50 best forecasts together with the kernel approximation of the empirical function.⁵ The forecasts produced by the 50 best models are not normally distributed, rather they exhibit fat tails, asymmetry and multimodality. Moreover, this measure of uncertainty is not increasing with the forecast horizon by construction. These

 $^{^{5}}$ The distribution approximation is produced using a smoothing density with normal kernel function.



characteristics differentiate these functions from the standard predictive densities. In our example looking at the 50 best forecast, our baseline projection would have not changed, as most of the models predicted a recession for the next two years. However, we would have been able to warn the policy maker about the high degree of uncertainty affecting our forecast.⁶

To conclude our example, figure 3 shows the box plot of all the forecasts produced by those models with an MSE smaller than the benchmark AR (179 models for 2009Q1, 183 for 2009Q2, 256 for 2009Q3, 265 for 2009Q4, and all the 267 models for the whole 2010). If we considered also figure 3, we would have refined our report to the policy maker by concluding that we predict negative average growth for the first three quarters of 2009, but positive growth for 2010 as suggested by the median forecast.

With this example we showed how it is possible to exploit the information delivered by a large number of factor models, and how this information can be used to measure forecast uncertainty. However, in order to validate our method we need to show that, if we repeat the same exercise on a period of low volatility, the forecasts produced by different factor models exhibit a smaller degree of heterogeneity.

In figure 4 to 6 we show pseudo real time forecasts produced as if we were at the end of 2006, well before the global recession. Figure 4 shows that the range of the forecasts for 2007 and 2008 is much smaller than those for 2009-2010, especially at the first $(0.37\ vs.\ 0.69)$ and the second $(0.16\ vs.\ 0.42)$ forecasting horizon. Similarly, figure 5 shows that although the forecasts are not normally distributed, their range is consistently smaller than the one obtained in the previous example (one step-ahead $0.62\ vs.\ 0.96$, two steps-ahead $0.34\ vs.\ 0.53$). Moreover, figure 6 shows that the interquartile range is quite small at all forecast horizons. Finally, table 10, which reports the standard deviation and the range of the forecasts, shows that forecast uncertainty increased a lot during the global recession.

5 Conclusions

In this paper we propose to exploit the heterogeneity of forecasts produced by different model specifications to measure (a special category of) model uncertainty. We present our approach by means of a pseudo real-time forecasting exercise on a large database of Italian data from 1982 to 2008. We estimate as many as 267 factor models by using all the main techniques available in the literature and we show that most of these estimated factor models beat a standard time series benchmark.

Our approach is simple and intuitive. It consists in selecting all the models that outperform some benchmark model, and then in approximating the empirical distribution of the forecast produced by these models. The moments higher than the first characterize this measure of uncertainty.

⁶It is also noteworthy that, in contrast with the results in table 9, among the 50 best available models some predicted a recovery for 2010, as it actually happened.



We present two historical examples, before and during the crisis. We show that the forecast distributions obtained by many models are asymmetric, multimodal, and with fat tails. As expected, our measure of uncertainty increased considerably during the recent global recession. A structural and general analysis of these empirical forecast distribution is left for future work.

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Tables

Table 3: Relative Mean Squared Error GDP

				GDI				
\overline{h}	1	2	3	4	5	6	7	8
DI	0.68	0.74	0.65	0.63	0.64	0.61	0.58	0.56
DI2	0.77	0.80	0.68	0.64	0.63	0.57	0.50	0.46
LDI	0.69	0.71	0.61	0.53	0.61	0.63	0.55	0.51
DIB	0.73	0.80	0.70	0.68	0.66	0.66	0.63	0.64
DI2B	0.89	0.93	0.76	0.69	0.65	0.63	0.60	0.60
$\mathrm{DF}_{m{a}}$	0.81	0.76	0.80	0.80	0.81	0.82	0.79	0.80
DF_b	0.80	0.76	0.80	0.80	0.81	0.82	0.79	0.80

Each cell reports relative mean squared errors, which are computed relative to an AR model.



Table 4: Relative Mean Squared Error Labor Market

<u> </u>									
-	h	1	2	3	4	5	6	7	8
	DI	0.99	1.11	1.04	1.18	1.33	1.36	1.35	1.40
	DI2	1.20	1.57	1.22	1.34	1.46	1.45	1.40	1.35
	LDI	0.97	1.15	1.06	1.33	1.71	1.71	1.59	1.29
ur	DIB	1.32	1.43	1.21	1.40	1.48	1.37	1.33	1.33
	DIB2	1.24	2.06	1.42	1.46	1.71	1.65	1.43	1.44
	DF_a	1.44	1.74	1.67	1.60	1.62	1.50	1.40	1.34
	DF_b	1.38	1.74	1.66	1.59	1.62	1.49	1.39	1.34
	DI	0.85	0.74	0.68	0.68	0.81	0.95	0.96	0.95
	DI2	0.86	0.84	0.60	0.67	0.82	0.87	0.83	0.71
	LDI	0.79	0.76	0.66	0.60	0.91	1.03	1.01	0.93
\Box	DIB	0.91	0.79	0.71	0.77	0.88	0.95	0.93	0.83
	DIB2	0.90	0.75	0.66	0.73	0.81	0.89	0.86	0.81
	DF_a	0.85	0.71	0.65	0.69	0.75	0.82	0.78	0.70
	DF_b	0.83	0.69	0.63	0.69	0.74	0.80	0.75	0.67
*	DI	1.01	1.01	1.00	0.91	0.90	0.91	0.85	0.76
	DI2	1.01	1.02	0.98	0.81	0.66	0.71	0.53	0.40
81-1	LDI	1.03	1.02	1.04	0.89	1.00	1.00	0.89	0.77
L.aff	$_{ m DIB}$	1.06	1.12	1.07	0.99	1.12	1.06	0.94	0.87
	DIB2	1.04	1.09	1.05	1.04	0.96	0.88	0.62	0.45
	DF_a	1.12	1.29	1.14	1.04	1.16	1.10	0.98	0.98
	DF_b	1.07	1.20	1.06	1.02	1.11	1.02	0.94	0.93
	DI	0.91	0.82	0.83	0.89	0.95	1.03	1.06	1.10
	DI2	0.76	0.76	0.77	0.90	1.01	1.11	1.12	0.98
DS I	LDI	0.91	0.82	0.85	0.96	1.05	1.22	1.33	1.23
P.cons	DIB	0.92	1.00	0.97	1.01	1.07	1.15	1.04	1.13
ı	DIB2	0.78	0.76	0.92	1.08	1.12	1.14	1.11	1.02
	DF_a	0.91	0.90	0.98	1.08	1.04	1.04	1.06	1.08
	DF_b	0.90	0.89	0.97	1.08	1.03	1.04	1.05	1.08
	DI	0.54	0.47	0.63	0.82	0.97	1.07	1.03	1.06
	DI2	0.58	0.51	0.54	0.89	1.08	1.17	1.03	0.98
q	LDI	0.56	0.52	0.64	0.75	1.06	1.21	1.20	1.10
.H.	DIB	0.68	0.59	0.66	0.84	1.01	1.08	1.03	0.97
21-24	DIB2	0.56	0.47	0.57	0.84	0.99	1.06	0.97	0.91
	DF_a	0.68	0.63	0.75	0.87	1.03	1.17	1.09	1.06
	DF_b	0.67	0.63	0.75	0.87	1.03	1.16	1.07	1.05
-	DI	0.88	0.84	0.74	0.61	0.57	0.68	0.65	0.68
	DI2	0.95	0.97	0.68	0.62	0.59	0.64	0.59	0.60
2	LDI	0.87	0.90	0.73	0.63	0.70	0.68	0.56	0.63
L.serv	DIB	1.09	1.06	0.71	0.59	0.61	0.65	0.53	0.52
П	DIB2	1.10	1.05	0.72	0.61	0.56	0.63	0.65	0.61
	DF_a	0.88	0.79	0.62	0.59	0.51	0.58	0.53	0.52
	DF_b	0.86	0.78	0.61	0.59	0.51	0.57	0.53	0.51
	97 7664					-000 EW			0.000

Each cell reports relative mean squared errors, which are computed relative to an AR model. ur = Unemployment Rate; L = Employment; L.aff = Employment in agriculture and forestry; L.cons = Employment in Constructions; L.ind = Employment in Industry; L.serv = Employment in services.



Table 5: Relative Mean Squared Error Gross Value Added

0			GI	OSS V	uue A	ииеи			
anoning of the	h	1.	2	3	4	5	6	7	8
	DI	0.81	0.82	0.77	0.74	0.67	0.61	0.55	0.54
£.,	DI2	0.81	0.80	0.77	0.71	0.63	0.52	0.43	0.40
- I	LDI	0.89	0.82	0.77	0.62	0.54	0.57	0.55	0.46
ξĊ	DIB	0.97	0.98	0.86	0.83	0.73	0.73	0.63	0.64
GVA.clcr	DIB2	0.95	0.90	0.85	0.82	0.73	0.68	0.57	0.58
	DF_a	1.00	0.97	0.93	0.88	0.82	0.79	0.73	0.75
	DF_b	0.99	0.96	0.93	0.88	0.82	0.79	0.73	0.75
	DI	0.96	0.98	0.85	0.93	1.12	1.34	1.49	1.58
Ø	DI2	1.03	1.25	1.02	1.10	1.31	1.50	1.63	1.46
GVA.cons	LDI	0.96	1.04	0.82	1.15	1.59	1.81	1.90	1.34
∀ .	DIB	0.97	0.90	0.90	1.06	1.13	1.30	1.36	1.37
25	DIB2	1.16	1.45	0.99	1.16	1.27	1.44	1.38	1.46
0	DF_a	0.92	0.88	0.89	1.04	1.13	1.24	1.28	1.29
,	DF_b	0.92	0.88	0.87	1.02	1.12	1.23	1.27	1.28
,	DI	0.77	0.79	0.80	0.72	0.68	0.66	0.65	0.63
ಲ್	DI2	0.80	0.92	0.81	0.74	0.70	0.66	0.63	0.58
GVA.indLc	LDI	0.71	0.75	0.68	0.57	0.57	0.61	0.59	0.56
.II.	DIB	0.86	0.85	0.84	0.77	0.75	0.72	0.69	0.71
\leq	DIB2	0.82	0.93	0.86	0.76	0.70	0.66	0.64	0.63
U	DF_a	0.92	0.85	0.88	0.85	0.83	0.84	0.81	0.82
	DF_b	0.87	0.84	0.88	0.85	0.83	0.84	0.81	0.81
	DI	0.60	0.64	0.52	0.55	0.52	0.47	0.41	0.40
4	DI2	0.50	0.64	0.58	0.62	0.56	0.49	0.42	0.41
GVA.serv	LDI	0.59	0.56	0.50	0.49	0.56	0.53	0.39	0.43
	DIB	0.58	0.66	0.55	0.63	0.59	0.58	0.56	0.59
2	DIB2	0.51	0.60	0.67	0.66	0.62	0.59	0.56	0.54
)	DF_a	0.55	0.61	0.65	0.77	0.77	0.78	0.73	0.76
	DF_b	0.55	0.60	0.64	0.76	0.76	0.77	0.73	0.76

Each cell reports relative mean squared errors, which are computed relative to an AR model. GVA.clcr = Gross Value Added in com., lodging, catering& rep; GVA.cons = Gross Value Added in Construction; GVA.indLc = Gross Value Added in Industry less Construction; GVA.serv = Gross Value Added in Services.



Table 6: Relative Mean Squared Error Consumption

				00760	arropou	770			
	h	1.	2	3	4	5	6	7	8
	DI	0.72	0.91	0.52	0.36	0.23	0.22	0.23	0.28
	DI2	0.87	0.97	0.51	0.50	0.37	0.36	0.30	0.28
	LDI	0.74	0.74	0.68	0.45	0.36	0.30	0.22	0.30
C	DIB	0.69	0.70	0.50	0.45	0.42	0.43	0.44	0.48
	DIB2	0.69	0.70	0.48	0.52	0.56	0.60	0.62	0.67
	DF_a	0.77	0.80	0.72	0.69	0.66	0.67	0.67	0.72
	DF_b	0.76	0.79	0.71	0.68	0.65	0.66	0.66	0.71
	DI	1.03	1.23	1.01	0.70	0.64	0.65	0.63	0.75
	DI2	1.12	1.24	1.04	0.63	0.57	0.54	0.53	0.69
\sim	LDI	1.19	1.14	0.95	0.50	0.60	0.67	0.57	0.71
E:	DIB	1.05	1.15	0.76	0.64	0.64	0.68	0.72	0.83
_	DIB2	1.23	1.35	0.75	0.62	0.66	0.75	0.83	1.02
	DF_a	1.09	1.11	0.83	0.74	0.74	0.77	0.78	0.84
	DF_b	1.08	1.10	0.83	0.73	0.74	0.77	0.77	0.84
	DI	0.73	0.66	0.75	0.73	0.66	0.65	0.67	0.72
	DI2	0.76	0.69	0.76	0.69	0.69	0.71	0.72	0.71
Q	LDI	0.77	0.67	0.78	0.85	0.76	0.79	0.73	0.81
nonD	DIB	0.80	0.71	0.79	0.78	0.78	0.77	0.83	0.83
\ddot{c}	DIB2	0.84	0.69	0.70	0.74	0.79	0.79	0.89	0.87
	DF_a	0.79	0.65	0.75	0.86	0.85	0.85	0.84	0.81
	DF_b	0.80	0.64	0.75	0.85	0.85	0.85	0.83	0.81
	DI	0.53	0.54	0.39	0.43	0.40	0.39	0.36	0.35
	DI2	0.48	0.59	0.38	0.43	0.43	0.44	0.41	0.40
Ü	LDI	0.61	0.78	0.44	0.36	0.34	0.44	0.38	0.35
.semiL	DIB	0.82	0.83	0.50	0.58	0.54	0.59	0.49	0.56
Ë	DIB2	0.80	0.86	0.54	0.56	0.56	0.57	0.51	0.59
888	DF_a	0.93	0.91	0.74	0.73	0.70	0.68	0.63	0.69
	DF_b	0.87	0.88	0.74	0.73	0.70	0.68	0.63	0.68
	DI	0.79	0.78	0.60	0.45	0.39	0.30	0.27	0.27
	DI2	0.77	0.59	0.54	0.48	0.46	0.33	0.31	0.29
5	LDI	0.83	0.81	0.61	0.48	0.39	0.46	0.34	0.34
serv.	DIB	0.81	0.74	0.61	0.50	0.45	0.41	0.45	0.49
C	DIB2	0.83	0.74	0.58	0.50	0.49	0.41	0.42	0.44
	DF_a	0.90	0.82	0.78	0.74	0.74	0.70	0.64	0.65
	DF_b	0.88	0.79	0.76	0.71	0.71	0.67	0.63	0.63
	Drb	0.00	0.79	0.76	0.71	0.71	0.67	0.03	0.0

Each cell reports relative mean squared errors, which are computed relative to an AR model. C. = Consumption; C.D = Consumption of Durable Goods; C.nonD = Consumption of Non Durable Goods; C.semiD = Consumption of Semi-Durable Goods; C.serv = Consumption of services.



Table 7: Relative Mean Squared Error Investments

	202001001001001001001001001	xaaaaaxaaaaa	***********	176068	итен	5			
,	h	1	2	3	4	5	6	7	8
	DI	0.74	0.75	0.51	0.57	0.67	0.73	0.74	0.80
	DI2	0.92	1.06	0.74	0.66	0.76	0.76	0.74	0.78
Ē	LDI	0.81	0.74	0.50	0.51	0.70	0.78	0.76	0.60
GFCF	DIB	0.79	0.73	0.63	0.67	0.73	0.74	0.76	0.82
Ç	DIB2	0.88	0.90	0.77	0.73	0.78	0.80	0.79	0.84
	DF_a	0.80	0.71	0.69	0.74	0.83	0.87	0.88	0.93
	DF_b	0.80	0.71	0.69	0.74	0.82	0.87	0.88	0.93
	DI	0.90	0.81	0.72	0.81	1.02	1.20	1.28	1.21
$\bar{\omega}$	DI2	0.98	0.98	0.83	0.91	1.08	1.25	1.38	1.39
${ m GFCF}_{ m cons}$	LDI	0.88	0.88	0.72	1.00	1.31	1.42	1.51	1.06
Ĕ	DIB	0.92	0.82	0.79	0.94	1.05	1.09	1.29	1.32
Ĕ	DIB2	0.94	0.89	0.85	0.92	1.15	1.26	1.48	1.53
0	DF_a	0.91	0.84	0.81	0.92	1.02	1.10	1.13	1.18
	DF_b	0.90	0.83	0.80	0.91	1.01	1.10	1.13	1.17
	DI	0.73	0.78	0.55	0.55	0.57	0.61	0.62	0.67
۲.	DI2	0.76	0.94	0.71	0.60	0.62	0.60	0.59	0.60
${ m GFCFmach}$	LDI	0.75	0.75	0.46	0.38	0.49	0.57	0.57	0.50
	DIB	0.77	0.76	0.61	0.62	0.64	0.62	0.67	0.73
F	DIB2	0.89	0.89	0.68	0.61	0.63	0.64	0.66	0.72
Q	DF_a	0.73	0.70	0.67	0.68	0.71	0.73	0.76	0.83
	DF_b	0.74	0.70	0.66	0.68	0.71	0.73	0.76	0.83
	DI	0.66	0.60	0.34	0.49	0.49	0.46	0.47	0.54
133	DI2	0.77	1.00	0.55	0.65	0.63	0.57	0.53	0.56
ran	LDI	0.67	0.67	0.47	0.53	0.54	0.57	0.59	0.53
GFCFtrans	DIB	0.73	0.68	0.50	0.66	0.64	0.60	0.59	0.57
Ę.	DIB2	0.79	0.81	0.64	0.72	0.70	0.72	0.68	0.75
Ŋ	DF_a	0.77	0.76	0.68	0.81	0.87	0.91	0.87	0.89
	DF_b	0.76	0.76	0.68	0.81	0.87	0.91	0.86	0.89
15000	8 2011	20 (3)		(0)	185 (De	errones:	60	850s (050)	200504

Each cell reports relative mean squared errors, which are computed relative to an AR model. GFCF = Gross Fixed Capital Formation; GFCFcons = Gross Fixed Capital Formation in Construction; GFCFmach = Gross Fixed Capital Formation in Machinery and Equipment; GFCFtrans = Gross Fixed Capital Formation in Transport.

Table 8: Number of Specifications with RMSE > 1 GDP

				905000 pN 465			
h	DI	DI2	LDI	DIB	DI2B	DF_a	DF_b
1	4	67	14	0	3	0	0
2	6	67	8	0	3	0	0
3	2	2	7	0	0	0	0
4	0	1	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	0	0	0	0	O	0	0

Each cell reports the number of specifications that does worse than the benchmark AR model. The total number of estimated specifications within each class of model are: DI = 80, DI2 = 80, LDI = 80, DIB = 6, DI2B= 6, DF $_{\alpha}$ = 3, DF $_{b}$ = 12.



Table 9: Pseudo Real Time Forecasts

Quarter	AR	DI	DI2	LDI	DIB	DI2B	DF_a	DF_b
2009Q1	-0.0125	-0.6368	-0.0674	-0.8648	-0.3724	-0.2250	-0.3668	-0.3745
2009Q2	0.0709	-0.4166	-0.0341	-0.5515	-0.1118	0.1021	-0.1756	-0.1829
2009Q3	0.1775	-0.3156	-0.2440	-0.3821	-0.1513	-0.0378	-0.0056	-0.0067
2009Q4	0.2297	-0.2393	-0.2183	-0.3848	-0.2316	-0.0352	0.0303	0.0274
2010Q1	0.3051	-0.2127	-0.1504	-0.2482	-0.1431	-0.0068	0.0683	0.0692
2010Q2	0.3369	-0.1982	-0.1275	-0.3574	-0.0671	-0.0031	0.1284	0.1288
2010Q3	0.3385	-0.1726	-0.1417	-0.2482	-0.0434	-0.0196	0.1398	0.1387
2010Q4	0.3351	-0.1352	-0.1092	-0.1842	-0.0031	0.0303	0.1748	0.1739

For each of the eight methodologies, we select the one that produces the minimum MSE. Bold entries are best for each forecast horizons.

Table 10: Variability of the Forecasts

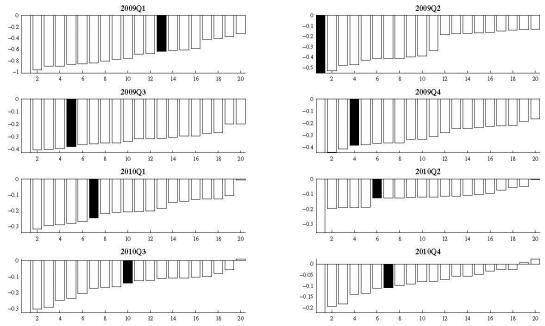
			Tab	ie 10.	varra	ourry .	of the	roreca	SiS		
		t_0	h	1	2	3	4	5	6	7	8
Standard		2006Q4	20best	0.14	0.04	0.10	0.07	0.09	0.08	0.06	0.12
	Deviation	90	50best	0.17	0.06	0.11	0.07	0.09	0.09	0.07	0.11
		20	all	0.18	0.09	0.11	0.12	0.12	0.11	0.10	0.10
		2008Q4	20best	0.20	0.15	0.06	0.09	0.08	0.07	0.09	0.06
		080	50best	0.20	0.15	0.08	0.10	0.09	0.12	0.10	0.08
g:		20	all	0.28	0.18	0.16	0.18	0.17	0.18	0.17	0.15
		24	20best	0.37	0.16	0.32	0.27	0.30	0.31	0.20	0.36
Range		2006Q4	50best	0.62	0.34	0.43	0.27	0.41	0.34	0.27	0.38
		20	all	0.23	0.13	0.14	0.17	0.13	0.14	0.14	0.09
		24	20best	0.69	0.42	0.23	0.28	0.34	0.35	0.34	0.25
		2008Q4	50best	0.96	0.53	0.31	0.41	0.36	0.46	0.41	0.38
		20	all	0.42	0.30	0.27	0.28	0.29	0.27	0.25	0.24

The rows 20best show standard deviation and range of the 20 models with minimum Mean Squared Error. The rows 50best show standard deviation and range of the 50 models with minimum MSE. Rows all show standard deviation and range of all the models that have an MSE smaller than the benchmark AR. Define \tilde{Y}^m_{t+h} the h step ahead forecast obtained with the m-th model, than for 20best and 50best "range" means $\tilde{Y}^{max}_{t+h} - \tilde{Y}^{min}_{t+h}$, while for all is the interquartile range.



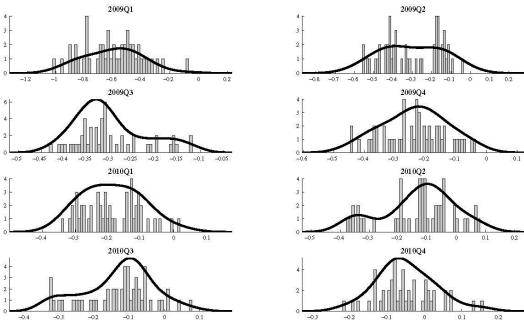
Graphs

Figure 1: 20 Best Forecast



Forecast are plotted from the lowest to the highest. They are not ranked in terms of MSE. The Black Bar is the Best Forecast





 ${\bf Figure~2:~} \textit{Distribution of the 50 Best Forecasts}$

These plots show Histograms of the forecasts produced with the $50\ best$ models together with the kernel approximation (black line) of the distribution

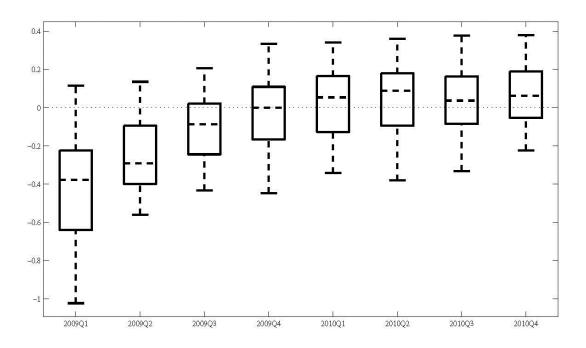


Figure 3: Box Plot



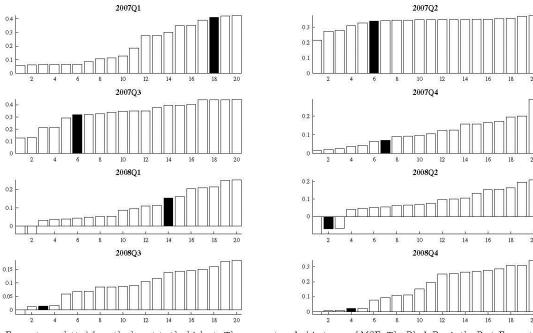


Figure 4: 20 Best Forecast

Forecast are plotted from the lowest to the highest. They are not ranked in terms of MSE. The Black Bar is the Best Forecast

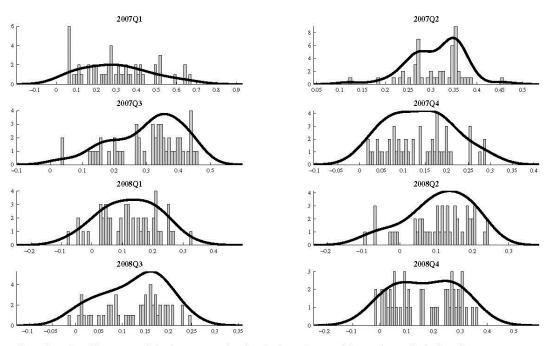
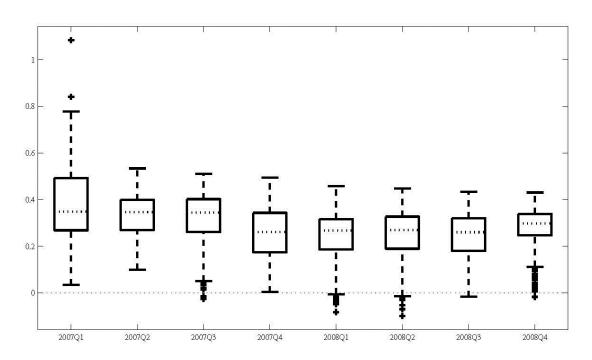


Figure 5: Distribution of the 50 Best Forecasts

These plots show Histograms of the forecasts produced with the $50\ best$ models together with the kernel approximation (black line) of the distribution



Figure 6: Box Plot





$\mbox{\bf Appendix}$ - Data Description and Data Treatment

N	C.	DSmnemonic	Name	Source	Unit	SA	F.	т.
1		ITGDPD	GDP	ISTAT	2000Mil€	1	Q	3
2		ITFNLUSED	Final Uses	ISTAT	2000Mil€	1	Q	3
3	Gross	ITGVACLCD	GVA - com., lodging, catering& rep	ISTAT	2000Mil€	1	Q	3
4	Domestic	ITGVACOND	GVA - construction	ISTAT	2000Mil€	1	Q	3
5	Product	ITGVAEDUD	GVA - ed.,health,oth.priv.& pub.svs.	ISTAT	2000Mil€	1	Q	3
6		ITGVAFMID	GVA - fuel & mining industries	ISTAT	2000Mil€	1	Q	3
7		ITGVAIXCD	GVA - industry excl. construction	ISTAT	2000Mil€	1	Q	3
8		ITGVASVSD	GVA - services	ISTAT	2000Mil€	1	Q	3
9		ITINVCHYD	CHANGE IN STOCKS	ISTAT	p_{t-1}	1	Q	0
10		ITCNPCDGD	PC - durable goods	ISTAT	2000Mil€	1	Q	3
11		ITCNPCFTD	PC - food alcohol & tobacco	ISTAT	2000Mil€	1	Q	3
12		ITCNPCFGD	PC - foreigners in italy	ISTAT	2000Mil€	î	$\vec{\mathbf{Q}}$	3
13		ITCNPCRAD	PC - italian residents abroad	ISTAT	2000Mil€	ī	Ž.	3
14	Consumption	ITCNPCNDD	PC - non-durable goods	ISTAT	2000Mil€	î	ര്	3
15	Consumption	ITCNPCNFD	PC - non-food	ISTAT	2000Mil€	i	QQ	3
16		ITCNPCSDD	PC - semi-durable goods	ISTAT	2000Mil€	1	ď	3
							Q	3
17		ITCNPCSVD	PC - services	ISTAT	2000Mil€	1		
18		ITCNPER.D	FDC - households	ISTAT	2000Mil€	1	Q	3
19		ITCNGOV.D	FDC - public	ISTAT	2000Mil€	1	Q	3
20		ITRVSTAXA	STATE BUDGET: TAX REVENUE	BdI	2000Bil€	2	M	3
21	Government	ITEXSCURA	STATE BUDGET: CURRENT EXPENDITURE	BdI	2000Bil€	2	M	3
22		ITEXSCAPA	STATE BUDGET: CAPITAL EXPENDITURE	BdI	2000Bil€	2	M	1
23		ITGOVBAAA	STATE BUDGET: BALANCE	BdI	2000Bil€	0	M	2
24		ITGFCFD	gross fixed capital formation	ISTAT	2000Mil€	1	Q	3
25	Investment	ITFCPCOND	GFCF - construction	ISTAT	2000Mil€	1	Q	3
26		ITFCPMCHD	GFCF - machinery & equipment	ISTAT	2000Mil€	1	Q	3
27		ITFCPTRND	GFCF - means of transport	ISTAT	2000Mil€	1	Q	3
28		ITEXPGD.D	exports of goods	ISTAT	2000Mil€	1	Q	3
29	Net	ITEXPSV.D	exports of services	ISTAT	2000Mil€	1	Q	3
30	Export	ITIMPGD.D	imports of goods	ISTAT	2000Mil€	î	Q	3
31	DAPOIC	ITIMPSV.D	imports of services	ISTAT	2000Mil€	1	Ğ.	3
32		ITULCAFFE		ISTAT	200071110	(3)		3
			ULC - agriculture forestry & fishing			1	Q	
33	********	ITULCCNSE	ULC - construction	ISTAT	2000=100	1	Q	3
34	Unit	ITULCOTHE	ULC - education, welfare, oth public & private svs	ISTAT		1	Q	3
35	Labor	ITULCCATE	ULC - hotels, trade, repair, public establishments	ISTAT	2000 = 100		QQ	3
36	Cost	ITLCOST.E	ULC - industry excluding construction	ISTAT	2000 = 100		Q	3
37		ITULCCAPE	ULC - credit & insurance	ISTAT	2000 = 100		Q	3
38		ITCNCTOTB	employee compensation	ISTAT	Mil€	1	Q	3
39		ITCNCAFFB	EC - agriculture, forestry & fishing	ISTAT	Mil€	1	Q	3
40		ITCNCCONB	EC - construction	ISTAT	Mil€	1	Q	3
41		ITCNCEDCB	EC - education, health, oth. priv. & pub. svs.	ISTAT	Mil€	1	Q	3
42	Employee	ITCNCFMIB	EC - fuel & mining industries	ISTAT	Mil€	1	0000000	3
43	Compensation	ITCNCHLTB	EC - health care	ISTAT	Mil€	1	Q	3
44		ITCNCHTCB	EC - hotels & pub. trnsp. & comm. repairs	ISTAT	Mil€	1	Q.	3
45		ITCNCIXCB	EC - industry excluding construction	ISTAT	Mil€	1	ര്	3
46		ITCNCSVRB	EC - services	ISTAT	Mil€	1	റ്	3
47		ITUN%TOTQ	unemployment rate	ISTAT	%	1	$\vec{\mathbf{Q}}$	2
48		ITCNETOTO	Employment	ISTAT	Thous.	1	Q	3
49		ITCNEAFFO		ISTAT	Thous.	1	Q	3
			E - agriculture forestry & fishing				3	3
50		ITCNECONO	E - construction	ISTAT	Thous.	1	ď	3
51		ITCNEEDUO	E - education health & other private & public svs.	ISTAT	Thous.	1	Q	3
52	Employment		E - fuel & mining industries	ISTAT	Thous.	1	Q	3
		ITCNEHLTO	E - health care	ISTAT	Thous.	1	da	3
53								
53 54		ITCNEHTCO	E - hotels & public trnsp. & communication repairs	ISTAT	Thous.	1	Q	3
53 54 55		ITCNEHTCO ITCNEINDO	E - industry	ISTAT	Thous.	1	Q Q	3
53 54		ITCNEHTCO					0000	3 3 3

NOTE: Variables 47 is backdated by using OECD Economic Outlook Data (DSMNEMONIC: ITOCFUNRQ). Variables 20-23 are deflated by using variable 77.



N	C.	DSmnemonic	Name	Source	Unit	$\mathbf{S}\mathbf{A}$	F.	Т
58		ITPRATE.	Discount Rate - Short Term euro repo rate	ECB	%	0	M	2
59		ECITLST	ITALY EURO-LIRE T/N (FT/ICAP/TR)	TR	%	0	M	2
60		ECITL1M	ITALY EURO-LIRE 1M (FT/ICAP/TR)	TR	%	0	M	2
61		ECITL3M	ITALY EURO-LIRE 3M (FT/ICAP/TR)	TR	%	0	M	2
62	Interest	ECITL6M	ITALY EURO-LIRE 6M (FT/ICAP/TR)	TR	%	0	M	2
63	Rates	ECITL1Y	ITALY EURO-LIRE 1 YR (FT/ICAP/TR)	TR	%	0	M	2
64		ITBI0257	EXPECTED GROSS MEAN YIELD (CCT)	BdI	%	0	M	2
65		ITQ61	GOVT BOND YIELD - LONGTERM	IFS	%	o	M	2
66		ITQ60B	MONEY MARKET RATE (FEDERAL FUNDS)	IFS	%	o	M	2
67		ITQ60C	TREASURY BILL RATE	IFS	%	0	M	2
68		11 @00C	ITQ61B ITQ60B	ML	%	0	M	2
10.00	Monorana	Trop N 6 s A		BdI		2		
69	Monetary	ITM1A	M1 - IT contribution to the euro area		Mil€		M	4
70	Aggregates	ITM3A	M3 - IT contribution to the euro area	BdI	Mil€	2	M	4
71		ITOCP009F	Consumer Price Index	MEI	2005 = 100	2	M	4
72		ITOCP041F	CPI - energy	MEI	2005 = 100	2	M	4
73		ITOCP042F	CPI - excluding food & energy	MEI	2005 = 100	2	M	4
74	Prices	ITOCP019F	CPI - food	MEI	2005 = 100	2	M	4
75		ITOCP057F	CPI - housing	MEI	2005 = 100	2	M	4
76		ITOCP064F	CPI - services less housing	MEI	2005 = 100	2	M	4
77		ITGDPIPDE	Implicit Price Deflator - GDP	ISTAT	2000=100	1	Q	4
78		ITIPDGOVE	Implicit Price Deflator - Gov.	ISTAT	2000=100	1	Q	4
79		ITOPRI35G	production of total industry (excluding construction)	MEI	2005=100	1	M	3
80	Industrial	ITOPRI49G	production of total manufactured consumer goods	MEI	2005=100	1	M	3
81	Production	ITOPRI61G	production of total manufactured intermediate goods	MEI		1	M	3
82	FIGURESION			MEI		1	M	3
AND ROLD	177 1	ITOPRI70G	production of total manufactured investment goods		2005=100	-17		
83	Exchange	ITOCC011	real effective exchange rate - cpi based	MEI	2005=100	2	M	3
84	Rates	ITOCC016	us cents to euro (ep)	MEI	\$/€	0	M	3
85		ITOSLI05E	total car registrations	MEI	2005 = 100	1	M	1
86		ITESP35GF	PPI: MANUFACTURE OF GAS	EUR	2005 = 100	2	M	4
87	Business	UKOILBREN	AVERAGE BRENT OIL PRICE	DEUK.	\$	0	M	4
88		ITOSP001F	share prices - ise mib storico	MEI	2005 = 100	0	M	3
89		ITOL1117Q	CLI - reference series	MEI	*	1	M	1
90	Confidence	ITOL0637Q	CLI - orderbooks or demand (fut. tend.)	MEI	%	1	M	1
91	Leading	ITOL0376Q	CLI - production - future tendency	MEI	%	1	M	1
92	Indicators	ITOL0577Q	CLI - volume net new orders (mfg.)	MEI	*	1	M	1
93	221000000000000000000000000000000000000	ITBIPCI.F	BdI Price Competitiveness Indicator - italy	BdI	1999=100	2	M	1
94		ITOBS083Q	BTS manufacturing - exports order books	MEI	%	1	M	1
95		ITOBS082Q	BTS manufacturing - future selling prices	MEI	%	1	M	1
96		ITOBS077Q		MEI	%	1	M	1
	S		BTS manufacturing - finished goods stocks					
97	Survey	ITOBS084Q	BTS manufacturing - future production	MEI	%	1	M	1
98		ITOBS078Q	BTS manufacturing - order books	MEI	%	1	M	1
99		ITCSECFTQ	ISAE CS economic climate index - future	ISAE	1980 = 100	1	M	1
100		ITCSECPRQ	ISAE CS economic climate index - present	ISAE	1980 = 100	1	M	1
101		BDGDPD	Ger - GDP	SBW	2000Bil€C	1	Q	3
102		FRGDPFD	Fra - GDP	INSEE	2000Mil€	1	Q	3
103		USGDPD	Us - GDP	BEA	2005Bil\$	1	Q	3
104		JPGDPD	Jpn - GDP	COJ	2005Bil€	1	Q	3
105		UKGDPMKTD	Uk - GDP	ONS	2005Mil£C	1	Q	3
106		BDCP7500F	Ger - CPI	SBW	1975 = 100	1	M	4
107		FRCONPRCF	Fra - CPI	INSEE	1998=100	1	M	4
108	Foreign	USCONPRCF	Us - CPI	BLS	**	ī	M	4
109	Countries	JPCPIEIAF	Jpn - CPI	MIAC	2005=100	î	M	4
	Countilles		Uk - CPI	ONS	2005 = 100 $2005 = 100$	1		4
110		UKD7BTQ.F					Q	
111		BDUN%TOTR	Ger - Unemployment Rate	DB	%	2	M	2
112		FRUN%TOTQ	Fra - Unemployment Rate	INSEE		1	Q	2
113		USUN%TOTQ	Us - Unemployment Rate	BLS	%	1	M	2
114		JPUN%TOTQ	Jpn - Unemployment Rate	MIAC	%	1	M	2
115		UKUN%TOTQ	Uk - Unemployment Rate	ONS	%	1	M	2
110		TICEDED THESE	FED Funds Rate	DDD	04	0	M	2
		USFEDFUN	FED Funds Rate	FED	%	U	TAT	- 2
116 117		UKPRATE.	BoE Base Rate	BoE	%	0	M	2

118 JPBANKR. PRIME RATE - LONG TERM BoJ % 0 M NOTE: Variable 101 is backdated by using OECD Economic Outlook Data (DSMNEMONIC: WGOCFGDPD), while variable 110 is backdated by OECD Main Economic Indicators Data (DSMNEMONIC: UKOCP009F):

* Actual number - RATIO TO TREND;

** 1982.1984=100.

List of Abbreviations

	Source	Transformations	Seasonally Adjustement
IFS EUR MEI ONS BdI FED BLS SBW MIAC BEA DB BoE BoJ COJ DEUK TR	Internation! Financial Statistics, IMF Eurostat OECD Main Economic Indicators OFFICE FOR NATIONAL STATISTICS Bank of Italy Federal Reserve Bank Bureau of Labor Statistics STATISTISCHES BUNDESAMT, WIESBADEN Ministry of Intarnal Affairs & Communications Bureau of Economic Analysis DEUTSCHE BUNDESBANK Bank of England Bank of Japan Cabinet Office, Japan Department of Energy, U.K Thomson Reuters	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0 Not Seasonally Adjusted 1 Seasonally Adjusted 2 SA with dummy variables regression

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